NRT/KS/19/5756

Bachelor of Arts (B.A.) First Semester Examination

MATHEMATICS (Calculus)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using ε–δ definition, show that :

$$\lim_{x\to 3}(2x^2+x)=21.$$ 6

(B) Examine the continuity of f(x) at x = 2,

where $f(x) = \begin{cases} \frac{|x-2|}{(x-2)} &, \text{ when } x \neq 2 \\ 0 &, \text{ when } x = 2 \end{cases}$. 6

OR

(C) If f is finitely derivable at a point x = a, then prove that f is continuous at x = a. Give a counter example to show that the converse is not true. 6

(D) If $y = (\sin^{-1}x)^2$, then prove that $(1-x^2)y_2 - xy_1 = 2$. Hence show that $(1-x^2)y_{n+2} - (2n+1)\, xy_{n+1} - n^2y_n = 0$. 6

UNIT—II

2. (A) Expand sin x in powers of $\left(x-\frac{\pi}{2}\right)$ upto three terms. 6

(B) If a curve is defined by the equation $x = f(t)$; $y = \phi(t)$, then prove that the radius of curvature is :

$$\rho = \frac{(x'^2+y'^2)^{3/2}}{x'y''-y'x''}$$

where(′) denotes the derivative with respect to t. 6

OR

(C) Find the asymptotes of the Cubic curve :

$y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x = 1$. 6

(D) Evaluate :

(i) $\lim_{x\to\infty}(x+e^x)^{2/x}$

(ii) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^3}$. 6

UNIT—III

3. (A) If $u = \log(x^2+y^2+z^2)$, then prove that :

$$x\frac{\partial^2 u}{\partial y\partial z} = y\frac{\partial^2 u}{\partial z\partial x} = z\frac{\partial^2 u}{\partial x\partial y}.$$ 6

(B) If z = f(x, y) is a homogenous function of degree n, then prove that :

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=n(n-1)z\,.$$ 6

OR

(C) If $u = x^2 - y^2$, $x = 2r - 3s$, $y = -r + 8s - 5$

find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$. 6

(D) If x + y + z = u, y + z = uv, z = uvw, find the value of the Jacobian of x, y, z with respect to u, v, w. 6

UNIT—IV

4. (A) Evaluate : $\int \frac{x^2+2x+3}{\sqrt{x^2+x+1}}\,dx$. 6

(B) Evaluate : $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$; x > 1. 6

OR

(C) Prove that :

$$\int \operatorname{cosec}^n x\,dx=\frac{-\operatorname{cosec}^{n-2}x\cot x}{(n-1)}+\frac{(n-2)}{(n-1)}\int \operatorname{cosec}^{n-2}x\,dx$$

Hence, evaluate $\int \operatorname{cosec}^3 x\,dx$. 6

(D) Show that :

$$\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}=\frac{\pi^2}{2ab}\,.$$ 6

Question—V

5. (A) Show that the function f defined by f(x) = $(1+3x)^{\frac{1}{x}}$ where x ≠ 0, f(0) = e^3 is continuous for x = 0. 1½

(B) Show that f(n) = x | x | is derivable at the origin. 1½

(C) Expand sin x by using Maclaurin's theorem. 1½

(D) Find radius of curvature for s = 4a sin φ. 1½

(E) If x = r cosθ, y = r sinθ, find $\frac{\partial(x,y)}{\partial(r,\theta)}$. 1½

(F) Find $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$ for the function z = $\tan^{-1}(y/x)$. 1½

(G) Evaluate $\int \frac{dx}{\sqrt{x^2+2x+3}}$. 1½

(H) Show that $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx=\frac{\pi}{4}$. 1½