

KNT/KW/16/6051-D

Bachelor of Arts (B.A.) Part-I Semester—I Examination**MATHEMATICS****Optional Paper-I****(M₁ : Algebra and Trigonometry)**

Time : Three Hours]

[Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All the questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.**UNIT-I**

1. (A) Find the rank of matrix A by reducing it into the normal form, where

$$A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

6

(B) Show that the following system of linear equations is consistent and hence solve the system of linear equations :

$$x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6.$$

6

OR

(C) Find the eigen values of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Also find the eigen vector corresponding to only one eigen value.

6

(D) Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem and find A^{-1} .

6

UNIT-II

2. (A) Solve the equation $x^3 - 3x^2 + 4 = 0$, two of its roots being equal. 6
- (B) If α, β, γ be the roots of the cubic $x^3 + px^2 + qx + r = 0$, then calculate the values of the symmetric functions :
- (i) $\Sigma \alpha^2$
- (ii) $\Sigma \alpha^2 \beta^2$
- (iii) $\Sigma \frac{1}{\alpha}$ 6

OR

- (C) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method. 6
- (D) Solve the biquadratic equation $x^4 + 12x - 5 = 0$ by Ferrari's method. 6

UNIT-III

3. (A) If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$. Then prove that :
- $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$ and
- $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin (\alpha + \beta + \gamma)$. 6
- (B) Find all the values of $(1)^{\frac{1}{4}}$. 6

OR

- (C) Prove that :
- (i) If $\sinh y = x$ then $y = \sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1} \right)$ and
- (ii) If $\cosh y = x$ then $y = \cosh^{-1} x = \log \left(x + \sqrt{x^2 - 1} \right)$ 6
- (D) Prove that :
- (i) $\text{Log} (-x) = (2n + 1) \pi i + \log x$ and
- (ii) $\text{Log} (xi) = (2n + \frac{1}{2}) \pi i + \log x$. 6

UNIT-IV

4. (A) Show that $G = \{1, -1, i, -i\}$ is an abelian group of order 4 with respect to multiplication. 6
- (B) Prove that intersection of two subgroups of a group is a subgroup. Give an example to show that union of two subgroups is not necessarily a subgroup of a group. 6

OR

- (C) Prove that the order of a subgroup of finite group is a divisor of the order of the group. 6
- (D) (i) Find whether a permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix} \text{ is even or odd.}$$

- (ii) If $f = \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ b_1 & b_2 & b_3 & \dots & b_n \end{pmatrix}$, then prove that $f.f^{-1} = I$, where I is the identity permutation of degree n . 6

QUESTION-V

5. (A) Write the augmented matrix $[A : B]$ for the system $y + 2z = a$, $x + 2y + 3z = b$, $3x + y + z = c$. 1½
- (B) Give that characteristic equation of the matrix :
- $$A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix} \text{ is } A^2 - 2A - 3I = 0. \text{ Find } A^{-1}. \quad 1\frac{1}{2}$$
- (C) Form an equation whose roots are 1, 2, 3. 1½
- (D) Using Descarte's rule of signs show that an equation $x^3 - 9x^2 + 12x = 0$ has two positive real roots and one negative real root. 1½
- (E) Prove that $\cos(ix) = \cosh x$. 1½
- (F) Prove that $\log(-1) = \pi i$. 1½
- (G) Prove that the identity element of group is unique. 1½
- (H) Find all the right cosets of subgroup $H = \{1, -1\}$ in a multiplicative group $G = \{1, -1, i, -i\}$. 1½