Bachelor of Arts (B.A.) First Semester Examination

MATHEMATICS (M_2 : Calculus)

Optional Paper–2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using $\in - \delta$ definition of limit, show that :

$\lim_{x \to 1} (2x+3) = 5$. 6

(B) If $f(x) = \dfrac{x\, e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, \quad x \neq 0$

$= 0, \quad x = 0$

then show that f(x) is continuous but not derivable at x = 0. 6

OR

(C) Find the nth derivative of $y = \tan^{-1}\left(\dfrac{x}{a}\right)$. 6

(D) If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$, then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. 6

UNIT—II

2. (A) Expand $7 + x + 3x^3 + x^4 - x^5$ in powers of (x+2), by using Taylor's Theorem. 6

(B) Find the radius of curvature at any point t on the curve x = a cost, y = b sin t. 6

OR

(C) Find the asymptotes of the curve $x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 1 = 0$. 6

(D) Evaluate :

(i) $\lim_{x \to \frac{\pi}{2}} (\cos x)^{\frac{\pi}{2} - x}$

(ii) $\lim_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$. 6

UNIT—III

3. (A) If $U = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 u = \dfrac{-9}{(x+y+z)^2}$. 6

(B) If $x^x\ y^y\ z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$. 6

OR

(C) If $u = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\ \sin 2u = 0$. 6

(D) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_2 x_3}{x_3}$

then find the Jacobian of x_1, x_2, x_3 with respect to y_1, y_2, y_3. 6

UNIT—IV

4. (A) Evaluate : $\int_0^1 \frac{1-4x+2x^2}{\sqrt{2x-x^2}}dx$. 6

(B) Evaluate : $\int \frac{dx}{(x-1)\sqrt{x^2+1}}$. 6

OR

(C) If $\phi(n) = \int_0^{\pi/4} \tan^n x\ dx$, then prove that $\phi(n) + \phi(n-2) = \frac{1}{n-1}$. Hence evaluate $\phi(5)$. 6

(D) Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx$. 6

Question—V

5. (A) Is the function $f(x) = x.\ \sin \frac{1}{x}$, $x \neq 0$

$= 0$, $x = 0$

continuous at the origin ? If yes, prove it. 1½

(B) Find y_n, if $y = \sin (ax + b)$. 1½

(C) Expand a^x in powers of x. 1½

(D) Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$. 1½

(E) If $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x \partial y \partial z}$. 1½

(F) If $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$. 1½

(G) Evaluate $\int_0^1 x^2 (1-x)^{\frac{3}{2}}\ dx$. 1½

(H) Evaluate $\int \frac{1}{\sqrt{x^2+2x+5}}\ dx$. 1½