NRT/KS/19/5757

Bachelor of Arts (B.A.) First Semester Examination

STATISTICS (PROBABILITY THEORY)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) Define with an example :

 (i) A random experiment

 (ii) Mutually exclusive outcomes

 (iii) Equally likely outcomes

 (iv) Exhaustive outcomes.

 Give classical definition of probability. State its limitations. Explain the relative frequency approach to probability. 10

OR

 (E) State and prove addition law of probabilities for two events. Hence prove it for three events.

 (F) State the two Boole's inequalities and prove any one of them. 5+5

2. (A) Define partition of the sample space state and prove Bayes theorem.

 (B) If A and B are independent events then show that :

 (i) A and $\overline{B}$ are independent

 (ii) $\overline{A}$ and B are independent

 (iii) $\overline{A}$ and $\overline{B}$ are independent. 5+5

OR

 (E) Two different suppliers A and B provide a manufacturer with the same part. All supplies of this part are kept is a large bin. In the past 5% of the parts supplied by A and 9% of the parts supplied by B have been defective. A supplies 4 times as many parts as supplied by B. A randomly selected part from the bin is found to be non defective. Find the probability that it is supplied by A.

 (F) For n events $A_1, A_2, \ldots\ldots A_n$ define pairwise independence and mutual independence. Show that mutual independence implies pairwise independence but its converse is not true. Derive the expressions for total number of conditions for mutual independence of n events. 5+5

3. (A) Define the cumulative distribution function of a discrete random variable. If $p(x_i)$, i = 1, 2 ...... n represents the p.m.f. of a discrete r.v. and f(x) represents its c.d.f. then show that :

 $p(x_1) = F(x_1)$ and

 $p(x_i) = F(x_i) - F(x_{i-1})$ i = 2, 3 ...... n

 Explain how the c.d.f. represents the step function.

(B) For a discrete r.v. define :

(i) Probability mass function

(ii) Expected value.

Let p.m.f. of a r.v. be given by :

X	1	2	3	4
p(x)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Find :

(i) E(X)

(ii) E[3X + 7]

(iii) V(2X + 5) 5+5

OR

(E) The distribution function of a r.v. X is given :

$$F(x)=\begin{cases} 0 & x<1 \\ \frac{1}{3} & 1\le x<4 \\ \frac{1}{2} & 4\le x<6 \\ \frac{5}{6} & 6\le x<10 \\ 1 & x\ge 10 \end{cases}$$

Find :

(i) $P[X \ge 5]$

(ii) $P[X > 10]$

(iii) $P[2 \le X < 7]$

(F) If the density function of X equals :

$$f(x)=\begin{cases} c\,e^{-2x} & 0<x<\infty \\ 0 & \text{elsewhere} \end{cases}$$

Find :

(i) C

(ii) $P[X > 2]$

(iii) $P[4 < X < 6]$ 5+5

4. (A) Define :

(i) r^{th} raw moment

(ii) r^{th} central moment

of a r.v. X.

Derive the expression for r^{th} central moment in terms of raw moments. Hence obtain the expressions for second, third and fourth central moments in terms of raw moments. 10

OR

(E) Define :

(i) Mean

(ii) Median

(iii) Mode

of a r.v. X.

If X is a r.v. with p.m.f.

X	0	1	2	3
p(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Find mean, median and mode of X. 10

5. Answer any **TEN** questions of the following :

(A) Why is the following assignment of probabilities not permissible if A and B are mutually exclusive events :

$P(A) = 0.6, \; P(A \cap \overline{B}) = 0.5$.

(B) If $A \subset B$, then show that $P(A) \leq P(B)$.

(C) State the three axioms involved in the axiomatic approach of probability.

(D) State Chebyshev's inequality.

(E) State the multiplicative law of probabilities for three events A, B and C.

(F) If A and B are two independent events then show that :

$$P(A \cup B) = 1 - P(\overline{A}) \cdot P(\overline{B}).$$

(G) Define discrete random variable with example.

(H) Given E(X) = 3; V(X) = 16.

Find V(4X + 1) and $E(X^2)$.

(I) Define expectation of a continuous random variable.

(J) Define standard deviation of a r.v. X.

(K) Define probability generating function of a r.v. X.

(L) Define moment generating function of a r.v. X. 1×10=10