Bachelor of Arts (B.A.) First Semester Examination
STATISTICS (Probability Theory)
Optional Paper–1

Time : Three Hours] [Maximum Marks : 50

N.B. :—ALL questions are compulsory and carry equal marks.

1. (A) State and prove two Boole's inequalities.

   (B) Give classical definition of probability. Explain its limitations. 5+5

OR

   (E) State and prove addition law of probabilities for two events. Hence prove it for three events.

   (F) Define with an example :

   (i) Equally likely events

   (ii) Mutually exclusive events

   Write the sample space for the following random experiment :

   (i) The tossing of three coins

   (ii) Tossing of 2 dice

   (iii) Coin and die are tossed together. 5+5

2. (A) Define partition of the sample space. State and prove Bayes theorem. In 2019, there will be three candidates for the position of Chief Minister Mr. A, Mr. B and Mr. C whose chances of getting the appointment are in the proportion 4 : 2 : 3 respectively. The probability that Mr. A if selected will promote tourism in Maharashtra is 0.3 ; the probability of Mr. B and Mr. C doing the same are respectively 0.5 and 0.8. What is the probability that tourism will be promoted in the State in 2019 ? 10

OR

   (C) Define conditional probability. Show that it satisfies axioms of probability.

   (D) If A and B are independent events then show that A and $\overline{B}$ are also independent events.

   (E) Show that the number of conditions required for mutual independence of 'n' events is $2^n - n - 1$.

   (F) State and prove multiplicative law of probability for n events. 2½×4=10

3. (A) The p.m.f of a r.v. x is

X	0	1	2
p(x)	3c	4c	3c

   where C > 0.

Find :

(i) Determine the value of C

(ii) Compute $P(X < 1)$, $P(X < 2)$, $P(1 < X \leq 2)$

(iii) Describe the distribution function and draw its graph.

(B) Define mathematical expectation of a.r.v. State and prove its properties. 5+5

OR

(E) Define c d f of a random variable X. State and prove properties of c.d.f.. If X is a. r. v. with p.m.f.

$$p(x) = \frac{x}{15} \;:\; x = 1, 2, 3, 4, 5$$
$$= 0 \;, \quad \text{elsewhere}$$

(i) Find cumulative distribution function of X

(ii) Find $P[X = 1 \text{ or } X = 2]$

(iii) Find $P\left\{\frac{1}{2} < X < 5/2 \;/\; X > 1\right\}$

(iv) Plot the graph of p.m.f. and c.d.f. 10

4. (A) Define skewness of a probability distribution of a r.v.. Explain the types of skewness with the help of figures. Define any two measures of skewness.

Let X be a r.v. with p d f

$$f(x) = 2(1-x) \quad 0 < x < 1$$
$$= 0 \qquad \text{otherwise}$$

Find the first three raw moments about origin. Hence find μ_2, μ_3 and β_1. Comment on the skewness of the probability distribution of X. 10

OR

(E) Define the p.g.f. of a r.v. X. Show how mean and variance can be calculated from it.

(F) Define the m.g.f. of a r.v. X. Show that it generates moments about origin.

(G) Define :

(i) Mean deviation from mean

(ii) Standard deviation

(iii) β_1

(H) Define :

(i) r^{th} raw moment about origin

(ii) r^{th} central moment

of a probability distribution of a r.v.

State the expression for the rth central moment in terms of raw moments about origin.

2½×4=10

5. Solve any **ten** of the following :

(A) Define sure event

(B) A card is selected from the pack of cards. Find the probability that it is an ace.

(C) If A, B, C are three events. Express the following events in appropriate symbols :

(i) The event A occurs but not B.

(ii) Simultaneous occurrence of A, B and C.

(D) State Chebyshev's inequality.

(E) State the conditions for mutual independence of three events A, B and C.

(F) Two events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$. Are A and B independent events?

(G) Let X be a r.v. with p.d.f.

$f(x) = 2(1-x), \; 0 < x < 1$

$\quad = 0 \quad$ otherwise

Find $P(X > 0.5)$.

(H) Let X be a r.v. with p.d.f.

$f(x) = 4x^3, \; 0 < x < 1$

$\quad = 0 \quad$ elsewhere

Find E(X).

(I) Let X be a r.v. with E(x) = 2 and V(x) = 8. Find

(i) $E[4X + 5]$

(ii) $V(3X + 2]$

(J) Define quartiles for a continuous r.v. X.

(K) If p.d.f. of a r.v. X is

$f(x) = \frac{1}{2} \quad -1 < X < 1$

$\quad = 0 \quad$ elsewhere

find the median.

(L) Let X be a r.v. with p.m.f.

X	2	3	4
P(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Find mode of the distribution. 1×10=10