NRJ/KW/17/6865

Bachelor of Arts (B.A.) (Part—I) Second Semester Examination

STATISTICS

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— All questions are compulsory and carry equal marks.

1. (A) Define a Poisson variate. Show that a recurrence relation for central moments of this distribution is $\mu_{r+1} = \lambda\left[\frac{d\mu_r}{d\lambda} + r\mu_{r-1}\right]$. Hence find μ_2 and μ_3. Comment on skewness of this distribution. Show that sum of independent Poisson variates is also a Poisson variate. 10

OR

(E) Obtain m.g.f. of a r.v. X which follows Binomial distribution with parameters n and p. Find first three row moments of this distribution. Hence obtain μ_2 and μ_3. Find β_1 and comment on skewness of this distribution. 10

2. (A) Give the p.m.f. of Geometric distribution. Why is it called so ? State and prove its lack of memory property. 5

(B) (i) If the probability is 0.25 that a movie fan gets a movie ticket on any given day, what is the probability that he will get the ticket on the fourth day ?

(ii) What is the probability that an IRS auditor will catch only two income tax returns with illegitimate deductions, if she randomly selects five returns among 15 returns, of which nine contain illegitimate deductions. 5

OR

(E) Derive the p.m.f. of Negative binomial distribution. Find its m.g.f. Hence obtain its mean and variance and comment.

A student appears for an oral examination. Each question has 5 multiple choices of which only one option is correct. He continues to answer the questions until he answers five questions correctly. What is the probability that he finishes his examination at the end of 25th question ? 10

3. (A) Obtain the mode of Normal distribution.

(B) Prove that a linear combination of independent normal variates is also a normal variate.

(C) Find mean and variance of the discrete uniform distribution of a r.v. X; where x = 1, 2, ..., n.

(D) If X is a continuous r.v. which is uniformly distributed over interval [a, b] with mean 1 and variance 4/3 then find $P(X < 0)$. 2.5×4=10

OR

(E) Show that median of the Normal distribution is equal to its mean.

(F) Show that all odd-ordered central moments of Normal distribution are zero.

(G) Let X denote the number of points obtained when an unbiased die is thrown. Name the distribution of X. Find mean and variance of X.

(H) If subway trains run every half hour, find the probability that a man entering the station at a random time will have to wait for at least twenty minutes ? 2.5×4=10

4. (A) If a r.v. $X \sim \text{Exp}(\theta)$ then show that $E(X) > V(X)$.

(B) Show that m.g.f. of gamma variable with parameter λ is $(1 - t)^{-\lambda}$.

(C) State the p.d.f. of Beta distribution of first kind and obtain its mean.

(D) Show that the probability distribution of sum of n independent gamma variables with parameters $\lambda_1, \lambda_2, ..., \lambda_n$ also follows gamma distribution. 2.5×4=10

OR

(E) Show that the exponential distribution lacks memory.

(F) Show that mean is equal to variance for gamma distribution with parameter λ.

(G) Find harmonic mean for the Beta variable of first kind with parameters m and n.

(H) Obtain r^{th} order raw moment about origin for Beta distribution of second kind and hence find its mean. 2.5×4=10

5. Solve any **TEN** of the following questions :

(A) If a r.v. X is a Bernoulli variate with parameter p then find its variance.

(B) State mode of a Binomial distribution whose mean is 4 and variance is 3.

(C) Obtain the p.g.f. of a Poisson distribution.

(D) Write p.m.f. of hypergeometric distribution, specifying range of the variable.

(E) State the distribution of mean of n independent standard normal variates.

(F) If independent Bernoulli trials are performed with constant probability 'p' of success in each trial, name the probability distributions of the following random variables :

(i) A r.v. X denoting number of failures preceding first success

(ii) A r.v. X denoting exact number of successes in n trials.

(G) Write the p.d.f. of the uniform variate over the interval (0, 1). Find its mean.

(H) Define a standard normal variate and write its pdf.

(I) Find the cdf of an exponential variate.

(J) If r.v. X follows gamma distribution with two parameters and its mean = 6 and variance = 12 then find its parameters.

(K) Find mean of Beta distribution of 1^{st} kind with parameters m and n.

(L) If pmf of a r.v. X is given by

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x & , x = 1, 2, \ldots. \\ 0 & , \text{elsewhere} \end{cases}$$

Identify the probability distribution and state its parameter. 1×10=10