NRT/KS/19/5852

Bachelor of Arts (B.A.) Third Semester Examination

MATHEMATICS

(Advanced Calculus, Sequence and Series)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Expand $f(x, y) = x^2 - xy + y^2$ in powers of $(x - 2)$ and $(y - 3)$. 6

(B) If f and g are continuous in [a, b] and derivable in (a, b) with $g'(x) \neq 0, \forall x \in [a, b]$, then prove that there exists $c \in (a, b)$ such that :

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$ 6

OR

(C) Using Lagrange's mean value theorem, show that :

$$\frac{x}{x+1} < \log(1 + x) < x, \forall x > 0.$$ 6

(D) Let $f : R^2 \to R$ be defined by :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Show that $\lim\limits_{(x,y) \to (0,0)} f(x, y)$ does not exist. 6

UNIT—II

2. (A) Find the envelope of the curve $x \cos^3\alpha + y \sin^3\alpha = c$, where α is the parameter. 6

(B) Find the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the relation $a + b = c$, c being constant. 6

OR

(C) Discuss the maximum or minimum values of u given by $u = x^3 + y^3 - 3axy$. 6

(D) Determine the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. 6

UNIT—III

3. (A) If sequences $< x_n >$ and $< z_n >$ each converge to l if $x_n < y_n < z_n, \forall n \in N$, then prove that the sequence $< y_n >$ also converges to l. 6

(B) Prove that the sequence $< x_n >$, where $x_n = \frac{2n+7}{3n+2}$ is a monotonic increasing sequence. Further show that it is bounded and tends to the limit $\frac{2}{3}$. 6

OR

(C) Prove that a sequence $< x_n >$ converges if an only if it is a Cauchy sequence. 6

(D) Evaluate :

$$\lim_{n\to\infty}\left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3 \ldots\ldots\ldots \left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}}.$$ 6

UNIT—IV

4. (A) Test for convergence of the series $\Sigma\{(n^3 + 1)^{1/3} - n\}$ by using comparison test. 6

(B) Test the convergence of the series $\sum \frac{n^3+a}{2^n+a}$ by using Ratio test. 6

OR

(C) Test for convergence of the series $\sum \frac{(n+\sqrt{n})^n}{2^n.n^{n+1}}$ by using Cauchy's Root test. 6

(D) Test the convergence of an alternating series :

$$\frac{2}{1}-\frac{3}{2^2}+\frac{4}{3^2}-\frac{5}{4^2}+\ldots\ldots\ldots$$ 6

QUESTION—V

5. (A) If in the Cauchy's mean value theorem $f(x) = e^x$ and $F(x) = e^{-x}$ for all $x \in [a, b]$, then show that c is arithmetic mean between a and b. 1½

(B) Show that :

$$\lim_{x\to 0}\lim_{y\to 0} f(x, y) = \lim_{y\to 0}\lim_{x\to 0} f(x, y)$$

where $f(x, y) = \frac{x^2+y^2}{x+y}$. 1½

(C) Find the envelope of the curve $(x - \alpha)^2 + y^2 = 4\alpha$, α being the parameter. 1½

(D) Prove that (4/5, 6/5) is a saddle point of the function $z = x^2 + y^2 - 3xy + 2x$. 1½

(E) Show that the sequence $< n^2 >$ is bounded below by 1 and not bounded above. 1½

(F) Find $n_0 \in N$ such that :

$$\left| \frac{2n}{n+3} - 2 \right| < \frac{1}{5} \text{ for all } n \geq n_0 .$$ 1½

(G) Test the convergence of the series whose n^{th} term is $\frac{3n-1}{2^n}$. 1½

(H) Test the convergence of the series $\sum \left(1+\frac{1}{n}\right)^{n^2}$, by root test. 1½