Bachelor of Arts (B.A.) Third Semester Examination

MATHEMATICS (M_6 Differential Equations and Group Homomorphism)

Optional Paper–2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) If λj and λk are roots of the equation $J_n(\lambda a) = 0$, then prove that :

$$\int_0^a x J_n(\lambda jx)\, J_n(\lambda kx)\,dx = 0, \text{ for } j \neq k.$$ 6

(B) Prove that :

(i) $$J_{-3/2}(x) = \sqrt{\frac{2}{px}}\left(-\frac{\cos x}{x} - \sin x\right).$$

(ii) $$J_{3/2}(x) = \sqrt{\frac{2}{px}}\left(\frac{\sin x}{x} - \cos x\right).$$ 6

OR

(C) Prove that $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n P_n(x)$, where $|x| \leq 1$, $|z| < 1$. 6

(D) Prove that : $nP_n = (2n-1)\,xP_{n-1} - (n-1)P_{n-2}$, $n \geq 2$. 6

UNIT—II

2. (A) If $L[f(t)] = F(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(u)\,du$, provided $\lim_{t\to 0} \frac{f(t)}{t}$ exist.

Hence, show that : $L\left[\frac{\sin t}{t}\right] = \tan^{-1}\left(\frac{1}{s}\right)$. 6

(B) Find $L[J_0(t)]$ and hence show that : $\int_0^{\infty} J_0(t)\,dt = 1$, where $J_0(t)$ is a Bessel's function of zeroth order. 6

OR

(C) Find :

(i) Laplace transform of f(t) defined as $f(t) = \begin{cases} t/a, & 0<t<a \\ 1, & t>a \end{cases}$.

(ii) Inverse Laplace transform of $\frac{1}{s(s^2+16)}$. 6

(D) (i) Find Laplace transform of f(t) = $t^2 \cos 2t$.

(ii) Evaluate $L^{-1}\left[\frac{1}{(S+1)(S^2+1)}\right]$ by using convolution property. 6

UNIT—III

3. (A) Solve Y" + Y = cos 2t, given that Y(0) = 1 and Y'(0) = –2. 6

(B) If u(x,t) be a function defined for t > 0 and x $\in$ [a,b], show that :

(i) $L\left[\frac{\partial u}{\partial t}\right] = SU - u(x,0)$ and

(ii) $L\left[\frac{\partial^2 u}{\partial t^2}\right] = s^2U - su(x,0) - u_t(x,0)$, where U = U(x,s) = L{u(x,t)} and $u_t(x,0) = \frac{\partial u}{\partial t}$ at t = 0. 6

OR

(C) Find the Fourier transform of $e^{-|x|}$. 6

(D) Find the Fourier cosine transform of $x^n e^{-ax}$. 6

UNIT—IV

4. (A) Prove that every quotient group G/H of a given group G is a homomorphic image of group G. Also prove that Kernel of f is H. 6

(B) If f : G $\to$ G' is an isomorphism and the order of a $\in$ G is n, then prove that order of f(a) is also n. 6

OR

(C) Prove that a subgroup N of a group G is a normal sub-group of G if and only if each left coset of N in G is a right coset of N in G. 6

(D) (i) Prove that every cyclic group is abelian.

(ii) Show that every quotient group of an abelian group is abelian but converse is not true. 6

Question—V

5. (A) Prove that : $\frac{d}{dx}[J_n] = J_{n-1}(x) - \frac{n}{x} J_n(x)$. 1½

(B) Prove that : $P_3(x) = \frac{1}{2}(5x^3 - 3x)$, where $P_3(x)$ is Legendre 's Polynomial of degree 3. 1½

(C) Prove that : $L[t^n] = \frac{n!}{s^{n+1}}$ 1½

(D) Find $L^{-1}\left[\frac{1}{s^2 - 4s + 20}\right]$. 1½

(E) Prove that : $L\left[\frac{\partial u}{\partial x}\right] = \frac{du}{dx}$, where U = U(x,s) = L[u(x,t)]. 1½

(F) Define Fourier Transform of a function f. 1½

(G) If $f : G \rightarrow G'$ is an isomorphism and e and e' are identities in G and G' respectively, then prove that f(e) = e'. 1½

(H) Prove that every subgroup of an abelian group is normal. 1½