Bachelor of Arts (B.A.) Fifth Semester Examination

MATHEMATICS (ANALYSIS)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find Fourier series of the function :

$$f(x) = \begin{cases} \frac{-\pi}{2} & , \ -\pi \le x < 0; \\ \frac{\pi}{2} & , \ 0 \le x \le \pi \end{cases}$$ 6

(B) Find Fourier series expansion of :

$f(x) = |x|$ in $-\pi \le x \le \pi$. 6

OR

(C) Find sine series of $f(x) = x^2$ in $0 \le x \le \pi$. 6

(D) Find Fourier series of the function :

$$f(x) = \begin{cases} -3 & , \ -2 \le x < 0 \\ 3 & , \ 0 \le x \le 2 \end{cases}$$ 6

UNIT—II

2. (A) If P is a partition of [a, b] and f(x) is bounded with bounds m, M such that $m \le f(x) \le M$, then prove that :

$m(b - a) \le L(P,f,\alpha) \le U(P,f,\alpha) \le M(b - a)$ where α is monotonic increasing function on [a, b]. 6

(B) If f is monotonic on [a, b] and if α is continuous on [a, b], then prove that $f \in R(\alpha)$, where $\alpha(x)$ is monotonic increasing on [a,b]. 6

OR

(C) If P* is a refinement of P, then prove that $L(P,f,\alpha) \le L(P^*, f, \alpha)$. 6

(D) If $f \in R(\alpha)$ on [a, b] and $|f(x)| \le K$ on [a, b], then prove that :

$$\left| \int_a^b f \, d\alpha \right| \le K \{\alpha(b) - \alpha(a)\}.$$ 6

UNIT—III

3. (A) If f(z) = u + iv is analytic function and $z = re^{i\theta}$, then show that Cauchy-Riemann equations are : $\frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. 6

(B) If $u = x^2 - y^2$, $v = -\frac{y}{x^2+y^2}$, then show that both u and v satisfy Laplace's equation but u + iv is not analytic function of z. 6

OR

(C) Prove that $u = y^3 - 3x^2y$ is harmonic. Also find harmonic conjugate v and analytic function f(z) = u + iv. 6

(D) If f(z) = u + iv is an analytic function in D, prove that the curves u = const., v = const. form orthogonal families of curves. 6

UNIT—IV

4. (A) Prove that every bilinear transformation is a combination of translation, rotation, magnification and inversion. 6

(B) Determine the region R′ in w-plane corresponding to the region R in z-plane bounded by the lines x = 0, y = 0, x + y = 1 under the transformation $w = z\, e^{i\pi/4}$. 6

OR

(C) Prove that if there is only one fixed point z = p, then the bilinear transformation $w = \frac{az+b}{cz+d}$ can be put in normal form :

$$\frac{1}{w-p} = \frac{1}{z-p} + k.$$ 6

(D) Find the fixed points and normal form of $w = \frac{(2+i)z-2}{z+i}$. Show that the transformation is loxodromic. 6

Question—V

5. (A) Let $f(x) = \log\left(\frac{1-x}{1+x}\right)$, $\forall x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. Verify whether function f is even or odd. 1½

(B) Find Fourier series of f(x) = cos x in $-\pi \le x \le \pi$. 1½

(C) Define U(P, f, α) and L(P, f, α) where symbols have usual meaning. 1½

(D) Let $\alpha(x) = x$, $\forall x \in [a,b]$ be a monotonic increasing function then find $\sum_{i=1}^{n} \Delta\alpha_i$. 1½

(E) Show that the function $f(z) = x^2+iy^2$ is not analytic. 1½

(F) Show that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic. 1½

(G) Is the transform $w = \frac{z+2}{2z+4}$ bilinear ? Explain. 1½

(H) Find the fixed points of $w = \frac{z-1}{z+1}$. 1½