NRT/KS/19/5973

Bachelor of Arts (B.A.) Sixth Semester Examination

MATHEMATICS (Discrete Mathematics and Elementary Number Theory) (Optional Paper)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Let R be a binary relation on the set of all positive integers such that R = {(a, b)/(a – b) is divisible by a positive integer m. Prove that R is an equivalence relation. 6

(B) Define the followings :

(i) Complete lattice

(ii) Bounds of a lattice

(iii) Bounded lattice

(iv) Complemented lattice

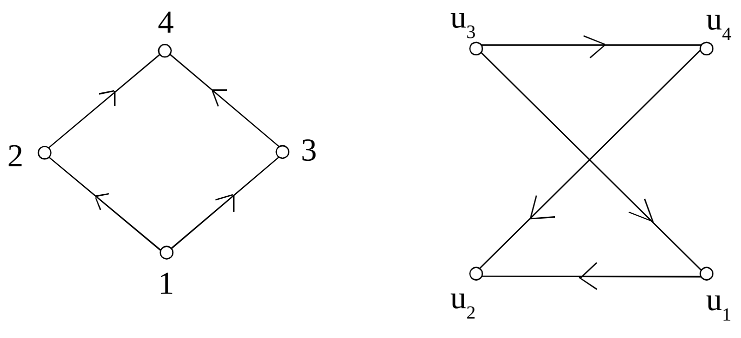
(v) Distributive lattice

(vi) Modular lattice. 6

OR

(C) Prove that every chain is a distributive lattice. 6

(D) Show that the following digraphs are isomorphic :



6

UNIT—II

2. (A) If g is the g.c.d. of b and i.e. (b, c) = g, then prove that there exists integers x_o and y_o such that $g = bx_o + ey_o$. 6

(B) Let f denote a polynomial with integral coefficients of $a \equiv b \pmod m$ then prove that $f(a) \equiv f(b) \pmod m$. 6

OR

(C) Find all the solutions of the linear congruence : $20x \equiv 35 \pmod{105}$. 6

(D) If (a, m) = 1, then prove that $a^{\phi(m)} = 1 \pmod m$. 6

UNIT—III

3. (A) Prove that 3 is a quadratic residue of 13 but a quadratic non-residue of 7. 6

(B) Is the congruence $x^2 \equiv -2 \pmod{59}$ solvable ? If so, then solve it. 6

OR

(C) Prove that $\sum_{j=1}^{p-1}\left(\frac{j}{p}\right)=0$, p being an odd prime. 6

(D) Evaluate :

$$\left(\frac{45}{71}\right).$$ 6

UNIT—IV

4. (A) Find all the solutions in positive integer : $5x + 3y = 52$. 6

(B) Solve the equation :

$$x + 2y + 3z = 10.$$ 6

OR

(C) Find all the Pythagorian Triples whose terms form an A.P. 6

(D) Construct Farey sequence F_5, if $F_1 = \left\{\frac{0}{1}, \frac{1}{1}\right\}$. 6

QUESTION—V

5. (A) Prove that in a lattice $(L, *, \oplus)$, $a * b = b * a$ and $a \oplus b = b \oplus a$. 1½

(B) Draw Hasse-diagram of {3, 5, 15}. 1½

(C) Find g.c.d. of 1819 and 3. 1½

(D) If $a \equiv b \pmod{m}$, then prove that $ac \equiv bc \pmod{mc}$ for $c > 0$. 1½

(E) Prove that $\left(\frac{a^2}{p}\right) = 1$, p prime. 1½

(F) Solve :

$$x^2 \equiv 81 \pmod{97}.$$ 1½

(G) Show that $2x + 3y = 4$ is solvable. 1½

(H) Define a Primitive Pythagorian Triple. 1½