NRT/KS/19/5975

Bachelor of Arts (B.A.) Sixth Semester Examination

MATHEMATICS (Special Theory of Relativity) (Optional Paper)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Derive general and simple Galilean transformations, considering two inertial frames S and S'. Also obtain their inverse transformations. 6

(B) Show that the three dimensional volume element dxdydz is not Lorentz invariant, but the four dimensional volume element dxdydzdt is Lorentz invariant. 6

OR

(C) Explain Lorentz-Fitz Gerald contraction hypothesis. Show that Lorentz-Fitz Gerald contraction hypothesis implies that there is no fringe shift in Michelson-Morley experiment. 6

(D) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformation. 6

UNIT—II

2. (A) Obtain the transformation equations for acceleration of a particle. 6

(B) Explain the phenomenon of time dilation in special theory of relativity. If a clock is moving with velocity c/3, then how much time it will loose in an hour ? 6

OR

(C) Prove that the simultaneity has only a relative and not an absolute meaning in special relativity. 6

(D) The space-time coordinates of two events measured in a frame S are $(x_o, 0, 0, x_o/c)$ and $(2x_o, 0, 0, x_o/2c)$. Find :

(i) the velocity of an inertial frame S' relative to S where these events are simultaneous.

(ii) the time t at which both events occur in the frame S'. 6

UNIT—III

3. (A) Define symmetric and skew symmetric contravariant tensors of rank 2. Show that any tensor of the rank 2 (covariant or contravariant) may be expressed as the sum of a symmetric and skew symmetric tensors. 6

(B) Show that Ars is a tensor if its inner product with an arbitrary mixed tensor B_t^s is a tensor. 6

OR

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(C) Find g and g^{ij} corresponding to the line element :

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$$

in terms of cylindrical coordinates ρ, ϕ, z. 6

(D) Define Four tensor, show that :

$$T'^{41} = \alpha^2 \left\{ -\frac{v}{c} T^{11} + T^{41} - \frac{v}{c} T^{44} + \frac{v^2}{c^2} T^{14} \right\}.$$ 6

UNIT—IV

4. (A) Obtain the mass energy equivalence $E = mc^2$. 6

(B) Define the four velocity and four acceleration of a particle. Show that four velocity and four acceleration are mutually orthogonal. 6

OR

(C) State the Maxwell's equations of electromagnetic theory in vacuum. Derive the wave equation for the propagation of the electric field strength $\overline{E}$ and magnetic field strength $\overline{H}$ in free space with velocity of light. 6

(D) Explain the term four potential and obtain the transformation equations of the electro-magnetic four potential vector under Lorentz transformations. 6

QUESTION—V

5. (A) State the fundamental postulates of special relativity. 1½

(B) Show that the circle $x'^2 + y'^2 = a^2$ in S' is measured to be an ellipse in S if S' moves with uniform velocity relative to S. 1½

(C) Suppose the half-life of a certain particle is 10^{-7} second, when it is at rest, calculate its half life when it is travelling with a speed of 0.8 c. 1½

(D) Derive Einstein's velocity addition law. 1½

(E) Prove that Kronecker delta δ^i_j is a mixed tensor of rank two. 1½

(F) Define time-like, space-like and light-like intervals. 1½

(G) Define four velocity and four acceleration. 1½

(H) Prove that :

$$p^2 = \frac{E^2}{c^2} - m_o^2 c^2.$$ 1½