B.E. All Branch Second Semester (C.B.S.) / B.E. (Fire Engineering) Second Semester

Applied Mathematics - II

P. Pages : 3 **NRJ/KW/17/4342/4998**

Time : Three Hours * 0 1 0 8 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) Prove that $\int_0^1 x^{n-1}\left(\log\frac{1}{x}\right)^{m-1} dx = \frac{\lceil m}{n^m}$. **5**

b) By differentiating under integral sign, evaluate the integral **7**

$$F(a)=\int_0^\infty \frac{e^{-ax}\sin x}{x}dx,$$

Hence show that $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$.

OR

2. a) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$. **6**

b) Obtain the root mean square value of $f(t)=3\sin 2t+4\cos 2t$ over the range $0 \le t \le \pi$. **6**

3. a) Trace the curve $3ay^2 = x(x-a)^2$. **6**

b) Find the area enclosed between the curve $y^2(2a-x)=x^3$ and its asymptote. **6**

OR

4. a) Find the area of the Cardioid $r = a(1+\cos\theta)$. **6**

b) Find the Perimeter of the astroid. **6**
$x^{2/3}+y^{2/3}=a^{2/3}$.

5. a) Evaluate: $\iint \frac{xy}{\sqrt{1-y^2}}\,dx\,dy$ over the positive quadrant of the circle $x^2+y^2=1$. **6**

b) Evaluate: $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **6**

c) Evaluate $\int_0^{a}\int_y^{a} \frac{x^2}{(x^2+y^2)^{3/2}}\,dy\,dx$ by changing it into polar coordinates. **6**

OR

6. a) Find the area outside the circle $r = a\cos\theta$ and inside the circle $r = 2a\cos\theta$. **6**

b) Find the mass of a plate in the shape of the curve **6**
$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1,$
the density being given by $\rho = \mu xy$.

c) Evaluate: $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx.$ **6**

7. a) Prove that:

i) $\hat{i}\times(\bar{a}\times\hat{i})+\hat{J}\times(\bar{a}\times\hat{J})+\hat{k}\times(\bar{a}\times\hat{k})=2\bar{a}$ **3**

ii) $[\bar{b}+\bar{c}\ \ \bar{c}+\bar{a}\ \ \bar{a}+\bar{b}] = 2[\bar{a}\ \bar{b}\ \bar{c}].$ **3**

b) A particle moves along the curve $x = t^3+1,\ y = t^2,\ z = 2t+5,$ where t is the time. Find the components of its velocity and acceleration at t = 1 in the direction i + j + 3k. **6**

c) Find the constants a and b such that the surface $ax^2-2byz=(a+4)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1, -1, 2). **6**

OR

8. a) Find the directional derivative of $\phi(x,y,z)=x^3-2y^2+4z^2$ at the point (1, 1, -1) in the direction of $2\hat{i}+\hat{J}-\hat{k}$. **6**
In what direction will the directional derivative be maximum and what is its magnitude.

b) Show that: **6**

i) Curl grad $\phi = 0$

ii) div curl $\bar{A} = 0$
where $\bar{A} = A_1\hat{i}+A_2\hat{J}+A_3\hat{k}$.

c) Show that $\overline{A} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find the scalar potential ϕ such that $\overline{A} = \nabla\phi$. **6**

9. If $\overline{A} = (y - 2x)i + (3x + 2y)j$, find the circulation of $\overline{A}$ about a circle C in the XY-plane with centre at origin and radius 2 if C is traverse in the positive direction. **7**

OR

10. Evaluate: $\int_C [(x^2 - \cosh y)dx + (y + \sin x)dy]$ **7**

by Green's theorem where C is the rectangle with vertices (0, 0), $(\pi, 0)$, $(\pi, 1)$, $(0, 1)$.

11. a) Fit the curve $y = ax^b$ to the following data by least square method. **7**

x:	1	2	3	4	5	6
y:	2.98	4.26	5.21	6.10	6.80	7.50

b) Find two missing terms from the following data. **6**

x	1	3	4	8	10
y	8	-	11	32	-

OR

12. a) Two lines of regression are given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$. if $6_x^2 = 12$, **7**

Find

i) The mean value of x and y
ii) Standard deviation of y
iii) The coefficient of correlation between x and y.

b) Solve: **6**

$$4y_{n+2} - 4y_{n+1} + y_n = \frac{n}{2^n}.$$

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