B.E. All Branches Second Semester (C.B.S.) / B.E. (Fire Engineering) Second Semester

Applied Mathematics - II

P. Pages : 3 **NIR/KW/18/3287/3941**

Time : Three Hours * 1 1 6 8 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. **6**

b) A rod of length 'a' is divided into two parts at random. Prove that the mean value of the sum of squares on these two segments is $\frac{2a^2}{3}$. **6**

OR

2. a) Evaluate $\int_0^{\infty} \frac{x^a}{a^x} dx$. **6**

b) By differentiating under integral sign, evaluate the integral. **6**
$\int_0^{\infty} \frac{e^{-ax}\sin x}{x} dx$ and hence show that $\int_0^{\infty} \frac{\sin x}{x} dx = \pi/2$.

3. a) Trace the curve $9ay^2 = (x-2a)(x-5a)^2$. **6**

b) Find the area included between the cardioid $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$. **6**

OR

4. a) Trace the curve $y^2 = x^2 - x^4$ and find area of its one loop. **6**

b) Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. **6**

5. a) Evaluate $\iint_R y\,dx\,dy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **6**

b) Evaluate $\iint_R (x^2 + y)\,dx\,dy$ by changing into polar form, where R is the region $x^2 + y^2 \le 1$. **6**

c) Evaluate by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\,dx\,dy$. **6**

OR

6. a) Find the area lying between the parabola $y = 4x - x^2$ and the line y = x. **6**

b) Evaluate $\iint_R r\,dr\,d\theta$ over the area of the curve $r = a(1+\cos\theta)$ above the initial line. **6**

c) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dz\,dx\,dy$. **6**

7. a) Show that the vector **6**
$(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})+(\bar{a}\times\bar{c})\times(\bar{d}\times\bar{b})+(\bar{a}\times\bar{d})\times(\bar{b}\times\bar{c})$
is parallel to the vector $\bar{a}$.

b) Find directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1, 1, -1) in the direction 2i+j-k. In what direction will the direction derivative be maximum? What is its magnitude? **6**

c) A vector field is given by $\bar{A} = (6xy + z^3)\,i + (3x^2 - 3)\,j + (3xz^2 - y)\,k$, prove that it is irrotational and hence find its scaler potential. **6**

OR

8. a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, find the components of its velocity and acceleration at t=1 in the direction of i-3j+2k. **6**

b) Find the angle between the tangents to the curve $\bar{r} = t^2 i - 2tj + t^3 k$ at the point t=1 and t=2. **6**

c) Prove that
i) curl grad $\phi = 0$ **3**
ii) div curl $\bar{A} = 0$ **3**

9. Use Green theorem in the plane to evaluate the integral **7**

$\int_c \left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy$ where C is the boundary in XY-Plane of the area enclosed by the x-axis and the semicircle $x^2 + y^2 = 1$ in the upper half of X Y Plane.

OR

10. A vector field is given by **7**

$F = (\sin y)i + x(1+\cos y)j$; evaluate $\int_C F \cdot dr$, where C is the circular path given by $x^2 + y^2 = a^2$, z=0.

11. a) Fit a parabola $y = a + bx^2$ for the following data by least square method. **7**

x:	1	2	3	4	5
y:	1.8	5.1	8.9	14.1	19.8

b) Find the missing term in the following data: **6**

x:	0	0.5	1	2.5	3
y:	–2	–0.375	3	–	19

OR

12. a) Two lines of regression are given by **7**

$x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ If $\sigma_x^2 = 12$ find (i) The mean of x and y (ii) the coefficient of correlation between x and y, and (iii) the standard deviation of y.

b) Solve the difference equation $y_{n+3} - 5y_{n+2} + 3y_{n+1} + 9y_n = 2^n + 3^n$. **6**

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