B.E. (Electronics Engineering / Electrical (Electronics & Power) Engineering / Electronics Telecommunication Engineering / Electronics Communication Engineering / Mechanical Engineering) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NRJ/KW/17/4352/4357/4362/4367**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$, then show that **6**

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{\bar{f}(s)}{s} \text{ and hence find } L\left\{\int_0^t u^2 e^{-u}\,du\right\}$$

b) Using convolution theorem, Find $L^{-1}\left\{\frac{S}{(S^2+1)^2}\right\}$ **6**

OR

2. a) Express $f(t) = \begin{cases} \cos t & , \quad 0 < t < \pi \\ \sin t & , \quad t > \pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **6**

b) Solve using Laplace transform method **6**

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin(t),\ y(0) = 0,\ y'(0) = 1$$

3. a) Obtain half range cosine series for $f(x) = 2x - 1$, in interval 0 < x< 1 and hence show that **6**

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$

b) Find Fourier sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$, $m > 0$ **6**

OR

4. a) Using Fourier integral, show that $\int_0^{\infty} \frac{\sin \pi\lambda \sin \lambda x}{1-\lambda^2} d\lambda = \begin{cases} \frac{\pi}{2}\sin x & , \quad 0 \le x \le \pi \\ 0 & , \quad x > \pi \end{cases}$ **6**

b) Find Fourier series for $f(x) = 2x - x^2$ in the interval $0 < x < 2$. **6**

5. Prove that shortest distance between two points is a straight line **6**

OR

6. Find the extremals of the functional $\int_1^2 \frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{x} dx$, given y(1) = 0, y(2) = 1. **6**

7. a) If $u + v = e^x [\cos y + \sin y]$ and $f(z)$ is an analytic function of z, find f(z), in terms of z. **6**

b) Evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is circle |z| = 3/2 by Cauchy's integral formula. **6**

c) Find the Laurent's series expansion of the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region **6**

i) $|z| < 2$ ii) $2 < |z| < 3$ iii) $|z| > 3$

OR

8. a) Show that $u = e^x \cos y + x^3 - 3xy^2$ is harmonic. Find its harmonic conjugate V. Hence find the analytic function f (z). **6**

b) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$, **6**

where C is a circle $|z| = 3$ by Cauchy residue theorem.

c) Evaluate the integral by contour integration $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$ **6**

9. a) Solve : $(mz - ny)p + (nx - \ell z)q = \ell y - mx$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ **7**

b) Solve : $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x \, \partial y^2} - 2\frac{\partial^3 z}{\partial y^3} = \cos(x+2y) - e^y(3+2x)$ **7**

OR

10. a) Solve using method of separation of variables, $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, given that $u=3e^{-y}-e^{-5y}$ when x = 0. **7**

b) Using Laplace transform method, solve **7**

$\frac{\partial Y}{\partial t}=\frac{\partial^2 Y}{\partial x^2}-4Y$,

Y(0, t) = 0 = Y(π, t),

Y(x, 0) = 6 sin x – 4 sin 2x

11. a) Find the modal matrix for the matrix. **6**

$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$

b) Use Sylvester's theorem to show that **6**

$3\tan A=(\tan 3)A$, where $A=\begin{bmatrix}-1 & 4\\ 2 & 1\end{bmatrix}$

c) Find whether the set of vectors are linearly dependent or otherwise, if linearly dependent find the relation between them **6**

$X_1=[1, 1, 1, 3],\ X_2=[1, 2, 3, 4],\ X_3=[2, 3, 4, 7]$

OR

12. a) Verify Cayley Hamilton theorem and express $A^6-4A^5+8A^4-12A^3+14A^2$ as a linear polynomial in A, if $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$ **6**

b) Solve $\frac{d^2y}{dt^2}+4y=0$, given $y=8,\ \frac{dy}{dt}=0$ when t = 0. **6**

c) Reduce the quadratic form **6**

$8x_1^2+7x_2^2+3x_3^2-12x_1x_2+4x_1x_3-8x_2x_3$ to canonical form by an orthogonal transformation.

****