B.E. (Mechanical Engineering) Fourth Semester (C.B.S.)

Applied Mathematics - IV

P. Pages : 3 **NJR/KS/18/4423**

Time : Three Hours * 0 6 1 3 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use normal distribution table is permitted.

1. a) Find the roots of $\cos x = 3x - 1$ Regula falsi method upto three decimal places. **6**

b) Solve the system of equation x + y + z = 1, 3x + y – 3z = 5, x – 2y – 5z = 10 by crout's method. **6**

OR

2. a) Solve the system of equation 5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20 by Gauss Seidal method. **6**

b) Derive the formula for Newton – Raphson method and hence compute $\sqrt{32}$ to four decimal places. **6**

3. a) Solve the differential equation **7**

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0,\ y(0) = 1,\ y'(0) = 0$$

for x = 0.1, h = 0.1 by Runge – Kutta method.

b) Solve $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$ $y(0.2) = 2.040$, $y(0.3) = 2.090$ **7**

Find y(0.4) and y(0.5) by Milne's Prediction – corrector method.

OR

4. a) Using Euler's modified method to solve the differential eq. $\frac{dy}{dx} = 1 - 2xy$ given y = 0 at x = 0 Hence find y at x = 0.4. **7**

b) Find largest eigen value and corresponding eigen vector for the matrix. **7**

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

5. a) If $Z(f(n)) = F(z)$ then prove that 6

$$Z(n f(n)) = -z F'(z) = -z \frac{d}{dz} F(z)$$

b) Solve difference equation. 6

$y_{n+2} - 3y_{n+1} + 2y_n = 3^n$ given that

$y_0 = 2,\ y_1 = -1$

OR

6. a) Find z-transform of $\cos(n\theta + \alpha)$ 6

b) Find $Z^{-1}\left\{\dfrac{2z^3 + 3z^2 + z}{(z^2+4)(z+1)}\right\}$ 6

7. a) Solve in series by Frobenius method $9x(1-x)\dfrac{d^2y}{dx^2} - 12\dfrac{dy}{dx} + 4y = 0$ 8

b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ 6

OR

8. a) State and prove the Rodrigue's formula 8

b) Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial. 6

9. a) The distribution function for the random variable X is defined as 6

$$F(x) = \begin{bmatrix} 1-e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{bmatrix}$$

Find :

i) Density function

ii) $P(x > 2)$

iii) $P(-3 < x < 4)$

b) The joint density function of R. V. X and Y defined as 8

$$f(x, y) = \begin{bmatrix} C(1-x^2-y^2), & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise} \end{bmatrix}$$

Find

i) Constant C

ii) Marginal density function of X and Y

iii) Check for independent.

OR

10. a) Find mathematical expectation for discrete random variable whose probability function is **4**

$$f(x) = \frac{x}{15}, \quad x = 1, 2, 3, 4, 5$$

b) Find moment generating for R. V. having density function **5**

$$f(x) = \begin{cases} e^{-2x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

also determine first four moment about origin.

c) Find characteristic function for R. V. X whose density function is **5**

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0, \ \alpha > 0 \\ 0, & \text{otherwise} \end{cases}$$

11. a) If the diameter of the ball bearing are normally distributed with mean 0.614 inches and standard deviation 0.0025 inches, determine the percentage of ball bearing with diameters. **7**

i) between 0.61 and 0.618 inches (both inclusive)

ii) greater than 0.617 inches

iii) equal to 0.615 inches

b) Out of 800 families with 4 children each, how many families would be expected to have **7**

i) 2 boys and 2 girls

ii) at least one boy

iii) no girl.

OR

12. a) Find the mean and variance of a continuous r.v.x is to be uniformly distributed in $a \leq x \leq b$, if its density function is **4**

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

b) Between the hours 2 pm and 4 pm the average number of phone calls per minute coming into the switch board of a company is 2.35. Find the probability that during one particular minute these will be at the most 2 phone calls. **5**

c) The auto correlation function for a stationary ergodic process is given by **5**

$$R_{xx}(\tau) = 25 + \frac{4}{1+16t^2}$$

Find the mean and variance of process x (t)

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