Bachelor of Science (B.Sc.) Semester—I (C.B.S.) Examination

MATHEMATICS (Algebra and Trigonometry)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative, solve each question in full or its alternative in full.

UNIT—I

1. (A) Find rank of the matrix A by reducing it into the normal form, where :

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$ 6

(B) Show that the equations :

$x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent and hence solve them. 6

OR

(C) Find characteristics roots of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

and find characteristic vector associated with the least value of the characteristic root. 6

(D) Show that the matrix $A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$ satisfies Cayley-Hamilton theorem. Hence find A^{-1}. 6

UNIT—II

2. (A) Solve the equation $x^3 - 10x^2 + 27x - 18 = 0$ given that one of its roots is double the other. 6

(B) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in arithmetic progression. Hence or otherwise solve the equation $x^3 - 6x^2 + 3x + 10 = 0$. 6

OR

(C) Solve $x^3 - 21x - 344 = 0$ by Cardon's method. 6

(D) Solve the biquadratic equation $x^4 - 3x^2 - 6x - 2 = 0$ using Ferrari's method. 6

UNIT—III

3. (A) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$, then prove that $x_1 . x_2 . x_3 ...... = -1$. 6

(B) Solve the equation :

$x^5 + 1 = 0$ using DeMoivre's theorem. 6

OR

(C) If $\cosh(\alpha + i\beta) = x + iy$, prove that :

(i) $\frac{x^2}{\cosh^2\alpha} + \frac{y^2}{\sinh^2\alpha} = 1$

(ii) $\frac{x^2}{\cos^2\beta} - \frac{y^2}{\sin^2\beta} = 1$ 6

(D) Separate into real and imaginary parts :

(i) $\cot(x + iy)$

(ii) $\log(x + iy)$. 6

UNIT—IV

4. (A) Let $(G, \circ)$ be a group. Then prove that :

(i) $(a^{-1})^{-1} = a, \forall a \in G$

(ii) $(a \circ b)^{-1} = b^{-1} \circ a^{-1} \; \forall a, b \in G$. 6

(B) Show that the set $G = \{1, -1, i, -i\}$ is an abelion group of order 4 under multiplication. 6

OR

(C) Prove that intersection of any two subgroups of a group is a subgroup.

Give an example to prove that union of two subgroups need not be a subgroup. 6

(D) Complete $a^{-1}ba$ for $a = (7, 5, 9)$, $b = (2, 1, 3)$. 6

Question-5

5. (A) Find eigen values of the matrix :

$$A = \begin{bmatrix} 6 & 4 \\ 1 & 2 \end{bmatrix}$$ 1½

(B) For a system of linear equations $x + 2y + z = 2$, $2x + 4y + 2z = 3$, $3x + 6y + 3z = 4$, find rank of the coefficient matrix. 1½

(C) Transform the equation :

$x^4 - 3x^2 + x^2 - x + 5 = 0$ into another whose roots shall be reciprocals of the roots of this equation. 1½

(D) If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then calculate the value of $\Sigma\alpha^2$. 1½

(E) Prove that :

$$\text{Log}(-x) = (2n + 1)\pi i + \log x$$ 1½

(F) Prove that :

$\cos(ix) = \cosh(x)$ and

$\sin(ix) = i \sinh(x)$ 1½

(G) Find disjoint right cosets of a subgroup $(Z_3, +)$ of group of integers $(Z, +)$. 1½

(H) Determine whether the permutation $f = (1\ 2\ 3\ 4\ 5)\ (1\ 2\ 3)\ (4\ 5)$ is old or even. 1½