**NRT/KS/19/2008**

**Bachelor of Science (B.Sc.) Semester—I Examination**

**MATHEMATICS (ALGEBRA AND TRIGONOMETRY)**

**Optional Paper—1**

Time : Three Hours] [Maximum Marks : 60

**Note :—** (1) Solve all the **five** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

**UNIT—I**

1\. (A) Reduce the Matrix $A=\begin{bmatrix}1 & 3 & 4 & 5\\ 1 & 2 & 6 & 7\\ 1 & 5 & 0 & 1\end{bmatrix}$

into its normal form and find its rank. 6

(B) Investigate for what values of $\lambda$ and $\mu$ the equations :

$x + 2y + z = 8,$

$2x + y + 3z = 13,$

$3x + 4y - \lambda z = \mu$

have (i) no solution

(ii) a unique solution

and (iii) infinite number of solutions. 6

**OR**

(C) Find characteristic roots of the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

and find characteristic vectors associated with the least-value of the characteristic root. 6

(D) Verify Cayey-Hamilton theorem for the matrix $A=\begin{bmatrix}0 & 0 & 1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{bmatrix}$. 6

**UNIT—II**

2\. (A) The cubic equation $2x^3 - 9x^2 + 12x - b = 0$ has two equal roots. Find values of b and solve the equation completely. 6

(B) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 + 3x^2 + 5x + 7 = 0$, then find the values of symmetric functions :

(i) $\sum \alpha^2$

(ii) $\sum \alpha^2\beta$

(iii) $\sum \alpha^2\beta\gamma$. 6

**OR**

(C) Solve the reciprocal equation $2x^5 - 7x^4 - x^3 - x^2 - 7x + 2 = 0$ by reducing it into its standard form. 6

(D) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method. 6

## UNIT—III

3. (A) Prove DeMoivre's theorem :

$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$,

for every positive and negative integer n. 6

(B) Prove that :—

$$(a+ ib)^{m/n} + (a - ib)^{m/n} = 2(a^2 + b^2)^{m/2n} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right).$$ 6

**OR**

(C) Separate into real and imaginary parts :

(i) $\text{Cot}(\alpha + i\beta)$

(ii) $\log(x + iy)$. 6

(D) Prove that if $\tanh y = x$, then $y = \tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$. 6

## UNIT—IV

4. (A) Prove that the set G = {1, 2, 3, 4, 5, 6} is a finite abelian group of order 6 with respect to multiplication modulo 7. 6

(B) Let (G, o) be a group. Show that $(a \circ b)^{-1} = b^{-1} \circ a^{-1}\ \forall\ a, b \in G$. 6

**OR**

(C) Show that order of a subgroup H of a finite group G is a divisor of the order of the group G. 6

(D) Show that out of the n! permutations on n symbols, $\frac{n!}{2}$ are even permutations and $\frac{n!}{2}$ are odd permutations. 6

## Questions—V

5. (A) Use Cayley-Hamilton theorem to find $A^8$, if $A = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$. 1½

(B) Determine the solution of the system of equations :

$y + 2z = 0,$

$x + y + 2z = 3$

$3x + 3y + 6z = 9$ 1½

(C) Find the condition that the sum of two roots of the equation $x^3 - px^2 + qx - r = 0$ is zero. 1½

(D) Show that equation $x^4 - 2x^3 - 1 = 0$ has at least two imaginary roots. 1½

(E) Find all the values of $(-i)^{1/3}$. 1½

(F) Determine the general value of Log (–5). 1½

(G) Determine the identity element of a group (I, o) of all integers under the operation $a \circ b = a + b + 1$. 1½

(H) Find inverse of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 1 & 3 \end{pmatrix}$ and evaluate $f \circ f^{-1}$. 1½