

NRT/KS/19/2009

Bachelor of Science (B.Sc.) Semester-I Examination
MATHEMATICS (CALCULUS)
Optional Paper—2

Time : Three Hours]

[Maximum Marks : 60]

- N.B. :**— (1) Solve all **FIVE** questions.
 (2) All questions carry equal marks.
 (3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using ϵ - δ technique of limit of a function, show that $\lim_{x \rightarrow 1} 3x^2 + x = 4$. 6

(B) Examine the continuity of the function f defined as :

$$f(x) = \begin{cases} 2+x & , x \leq 1 \\ 4-x & , 1 < x \leq 2 \\ -2+3x-x^2 & , x > 2 \end{cases}$$

at the points $x = 1$ and $x = 2$. 6

OR

(C) Let $f(x) = x \cdot \sin(1/x)$, $x \neq 0$

$$= 0 \quad , \quad x = 0$$

Show that f is continuous but not differentiable at $x = 0$. 6

(D) If $y = a \cos(\log x) + b \sin(\log x)$, then show that :

$$x^2 y_{n+2} + (2n + 1)x y_{n+1} + (n^2 + 1)y_n = 0. 6$$

UNIT—II

2. (A) Prove by Maclaurin's theorem, that :

$$e^x \cdot \log_e(1+x) = x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{9x^5}{5!} + \dots \dots \dots 6$$

(B) Find the radius of curvature for the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$ at $t = \pi/2$. 6

OR

(C) Find the asymptotes of the curve :

$$y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x = 1. 6$$

(D) Evaluate :

(i) $\lim_{x \rightarrow 0} x \tan\left(\frac{\pi}{2} - x\right)$

(ii) $\lim_{x \rightarrow 0} (\operatorname{cosec} x)^{\frac{1}{\log x}}$. 6

UNIT—III

3. (A) If $u = \log_e \sqrt{x^2 + y^2 + z^2}$, then prove that :

$$(x^2 + y^2 + z^2) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 1. \quad 6$$

(B) If $z = f(x, y)$ and $x = e^u - e^{-v}$, $y = e^{-u} + e^v$, then prove that :

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}. \quad 6$$

OR

(C) If z is a homogeneous function of x and y of degree n , then prove that :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz. \quad 6$$

(D) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$ and $z = r \cos \theta$, then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta. \quad 6$$

UNIT—IV

4. (A) Evaluate :

$$\int \frac{2x+5}{\sqrt{x^2+3x+1}} dx. \quad 6$$

(B) Show that :

$$\int_0^1 \frac{(1-4x+2x^2)}{\sqrt{2x-x^2}} dx = 0. \quad 6$$

OR

(C) Prove that :

$$\int \sec^n x dx = \frac{\sec^{n-2} x \cdot \tan x}{n-1} + \frac{(n-2)}{(n-1)} \int \sec^{n-2} x dx$$

and hence evaluate $\int \sec^6 x dx.$ 6

(D) Evaluate :

$$\int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}. \quad 6$$

QUESTION—V

5. (A) Show that $f(x) = \frac{1}{1-e^{1/x}}$, $x \neq 0$ has a simple discontinuity at $x = 0$. 1½

(B) If $y = \sin(ax + b)$, then prove that :

$$y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right). \quad \text{1½}$$

(C) Expand $\log x$ in powers of $(x - 1)$ upto the terms in x^2 . 1½

(D) Find the radius of curvature of the curve $s = c \log \sec \psi$. 1½

(E) If $z = \sin xy$ and $x = 2t + 5$, $y = 3t^2$, find $\frac{dz}{dt}$. 1½

(F) If $u = 2x + 3y$; $v = 5x + 6y$, then find $\frac{\partial(u, v)}{\partial(x, y)}$. 1½

(G) Evaluate :

$$\int_0^{\pi/2} \sin^5 x \cdot \cos^4 x \, dx. \quad \text{1½}$$

(H) Find :

$$\int \frac{dx}{\sqrt{x^2 - 2x + 5}}. \quad \text{1½}$$