Bachelor of Science (B.Sc.) Semester—I (C.B.S.) Examination

MATHEMATICS

(Algebra and Trigonometry)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1\. (A) Find rank of the Matrix A by reducing it into the normal form, where :

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}.$$ 6

(B) Investigate for what value of λ and μ, the simultaneous equations $x + 2y + z = 8$, $2x + y + 3z = 1$, $3x + 4y - \lambda z = \mu$ have (i) no solution (ii) unique solution and (iii) infinitely many solutions. 6

OR

(C) Find eigen values and the corresponding eigen vectors with respect to the greatest eigen value of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 6

(D) Verify Cayley-Hamilton's theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$. 6

UNIT—II

2\. (A) If α, β, γ are the roots of cubic equation $x^3 + px^2 + qx + r = 0$, then calculate the value of symmetric functions (i) $\Sigma\alpha^2$ (ii) $\Sigma\alpha^2\beta$ (iii) $\Sigma\alpha^2\beta^2$. 6

(B) Solve the reciprocal equation $x^2 - 5x^4 + 6x^3 - 6x^2 + 5x - 1 = 0$ by reducing it to the standard form. 6

OR

(C) Solve the cubic equation $x^3 - 6x - 9 = 0$ by Cardon's method. 6

(D) Solve the equation $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$ by Ferrari's method. 6

UNIT—III

3\. (A) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that :

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$. 6

(B) Find all the values of $(1)^{1/n}$. Show that these n roots form a series in geometric progression. 6

OR

(C) Separate log sin (x + iy) into real and imaginary parts. 6

(D) If cosh y = x, then prove that :

$$y = \cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right].$$ 6

UNIT—IV

4. (A) Prove that the set G = {1, 2, 3, 4} forms an abelian group with respect to multiplication modulo 5. 6

(B) In a group (G, 0); prove that :

(i) Identity element of G is unique

(ii) Inverse of every element of G is unique

(iii) $(a^{-1})^{-1} = a,\ \forall\ a \in G$. 6

OR

(C) Of the n! permutations on n symbols, prove that $\frac{n!}{2}$ are even permutations and $\frac{n!}{2}$ are odd permutations. 6

(D) Show that H = {3m/m $\in$ Z} is a subgroup of additive group of integers. Further write all the distinct right cosets of H in (Z, +). 6

Question–V

5. (A) If matrix A satisfies its characteristic equation $\lambda^3 - 3\lambda^2 + 2\lambda - 4 = 0$, then find A^{-1}. 1½

(B) Find eigen values of the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. 1½

(C) Form the cubic equation whose roots are $1, 1 + i\sqrt{3}$. 1½

(D) Using Descartes' rule of sign, investigate the nature of roots of the equation :

$$x^9 + x^7 - x^4 - 4x^3 - x^2 + 5 = 0.$$ 1½

(E) Express a complex number z = 1 + i in polar form. 1½

(F) Evaluate the value of $z = e^{in\pi} - e^{-in\pi}$. 1½

(G) Find whether H = {0, 1, –1} is a subgroup of additive group of integers. 1½

(H) Find index of H in G, when H = {1, –1} is a subgroup of multiplicative group G = {1, –1, i, –i}. 1½