NRT/KS/19/2024

Bachelor of Science (B.Sc.) Semester-I Examination

STATISTICS (PROBABILITY THEORY)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) Explain the three approaches to probability giving their merits and demerits.

If two dice are thrown, what is the probability that the sum is :

(a) Greater than 8,

(b) Neither 7 nor 11 ? 10

OR

(E) State additive law of probability for n events. Prove it for 3 events.

(F) (i) If A and B are any two events then show that :

$$P(\overline{A} \cap B) = P(B) - P(A \cap B).$$

(ii) If A_1, A_2, ... A_n are any 'n' events then prove that :

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$ 5+5

2. (A) Explain the concept of independence of two events giving one example.

If A and B are independent events then show that :

(i) A and $\overline{B}$ are independent.

(ii) $\overline{A}$ and B are independent.

(iii) $\overline{A}$ and $\overline{B}$ are independent.

(B) State and prove Bayes theorem. 5+5

OR

(E) For any three events A, B and C, show that :

(i) $P(A \cup B/C) = P(A/C) + P(B/C) - P[(A \cap B)/C]$

(ii) $P(A \cap \overline{B}/C) + P[(A \cap B)/C] = P(A/C)$

(F) Define mutual independence of n events. Show that the number of conditions to be satisfied for mutual independence of n events is $2^n - n - 1$.

An urn contains four tickets marked with numbers 112, 121, 211, 222 and one ticket is drawn at random. Let A_i (i = 1, 2, 3) be the event that i^{th} digit of the number of the ticket drawn is 1. Show that the events A_1, A_2, A_3 are pairwise independent but not mutually independent. 5+5

3. (A) Define with example :

(i) Discrete random variable

(ii) Probability distribution of a discrete r.v.

(iii) Expected value of a r.v.

(B) Define c.d.f. of a r.v. state and prove properties of c.d.f. 5+5

OR

(E) Let X be a random variable with p.d.f. given by :

$$f(x) = \begin{cases} cx^2 & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the constant C and CDF of X.

(b) Find E(X) and V(X).

(c) Find $P\left(X \geq \frac{1}{2}\right)$.

(F) Let X be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} 4x^3 & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) $P\left[X \leq \frac{2}{3} \middle/ X > \frac{1}{3}\right]$

(ii) $E[X^2 + 2X]$

(iii) V(2X + 5). 5+5

4. (A) Define probability generating function of a r.v. X and show that E(X) = P'(1). Also show that $V(X) = p''(1) + p'(1) - [p'(1)]^2$.

(B) Given the p.m.f. of a discrete r.v. X :

X	1	2	3
p(x)	0.7	0.2	0.1

Find :

(i) m.g.f. of X.

(ii) Obtain mean and variance of X using m.g.f.

(C) Define Skewness of the probability distribution of a r.v. Explain the types of skewness with the help of figures.

(D) Define Quartiles, Median and Mode for a continuous r.v. X. 2½×4=10

OR

(E) For a r.v. X, define :

(i) Mean

(ii) Mean deviation from mean

(iii) Standard deviation

(iv) Quartile deviation

(v) Coefficient of Skewness β_1.

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Let X be a r.v. with p.m.f.

X	2	3	4
p(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Find mean, mean deviation from mean, standard deviation and Karl Pearson's coefficient of skewness. 10

5. Solve any **TEN** of the following :

(A) Show that the probability of the complementary event $\overline{A}$ of A is given by $P(\overline{A}) = 1 - P(A)$.

(B) If the event $A \subset B$, then show that $P(A) < P(B)$.

(C) Define a random experiment with an example.

(D) If A, B, C are mutually independent events then show that $A \cup B$ and C are also independent.

(E) Define partition of the sample space with an example.

(F) Can two independent events be mutually exclusive ? Justify.

(G) A chemical can have life (in days) on the basis of following density function,

$$f(x) = \frac{1}{3x^5} \quad \text{if} \quad x \geq 1$$
$$= 0 \quad \text{if} \quad x < 1.$$

Find the probability that the chemical can have a life from 0 to 5 days.

(H) Three coins are tossed. Find the probability distribution of number of tails in three tosses.

(I) Give an example of discrete random variable.

(J) Define Kurtosis of a probability distribution.

(K) Define r^{th} raw moment and r^{th} central moment of a r.v. X.

(L) Define Bowley's coefficient of skewness. State its limits. 1×10=10