Bachelor of Science (B.Sc.) Semester—II Examination

MATHEMATICS (GEOMETRY, DIFFERENTIAL AND DIFFERENCE EQUATIONS)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Obtain the equation of the sphere which touches the sphere :

$$4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$$

at (1, 2, –2) and passes through the point (–1, 0, 0). 6

(B) Obtain the equation of the sphere having the circle :

$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the great circle. 6

OR

(C) Find the equation of a cone whose vertex is the point (1, –2, 1), axis is the line $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2}$ and semi-vertical angle is 60º. 6

(D) Find the equation of the right circular cylinder of radius 2 whose axis pass through the point (1, 0, 3) and has direction cosines proportional to (2, 3, 1). 6

UNIT—II

2. (A) Solve : $x\frac{dy}{dx} + y = x \log x$. 6

(B) Show that the differential equation :

$$(2xy^3 + y \cos x)dx + (3x^2y^2 + \sin x)dy = 0$$

is exact and hence solve it. 6

OR

(C) Solve : $x\frac{dy}{dx} + y = x^4y^3$ by reducing to the linear form. 6

(D) Solve : $3x^4p^2 - xp - y = 0$, where $p \equiv \frac{dy}{dx}$. 6

UNIT—III

3. (A) Solve : $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{2x}.\sin x$. 6

(B) Solve :

$$(x^2D^2 - xD + 2)y = x \log x, \text{ where } D \equiv \frac{d}{dx}.$$ 6

OR

(C) Solve $y^{(2)} + 4y = 4 \tan 2x$ by the method of variation of parameters. 6

(D) Solve $xy^{(2)} - (x - 2)y^{(1)} - 2y = x^3$ by using a known solution $y_1 = e^x$ included in its complementary function. 6

UNIT—IV

4. (A) From the difference equation by eliminating arbitrary constants A and B from :

$$y_n = A.3^n - B.5^n.$$ 6

(B) Solve : $u_{x+2} - 4u_{x+1} + 4ux = x^2 . 2^x$. 6

OR

(C) Solve the difference equation :

$$(E^2 - 5E + 6)y_n = 4^n(n^2 + n - 7).$$ 6

(D) Solve :

$$u_{x+2} - 2 \cos \alpha . u_{x+1} + u_x = \cos(\alpha x), \text{ where } \alpha \text{ is constant.}$$ 6

QUESTION-V

5. (A) Obtain the equation of the sphere described on the join of the points (2, 1, 1) and (5, 6, 9) as diameter. 1½

(B) Define right circular cone and right circular cylinder. 1½

(C) Solve : $xp^2 - yp + a = 0$, where $p \equiv \frac{d}{dx}$. 1½

(D) Find the integrating factor for the differential equation $y\,dx - x\,dy + x\,dx = 0$ by inspection and then solve it. 1½

(E) Solve : $(D^3 + 5D^2 - 5D - 1)y = 0$, where $D \equiv \frac{d}{dx}$. 1½

(F) Find the particular integral of :

$$(D^2 - 3D + 2)y = \cos(2x + 5), \text{ where } D \equiv \frac{d}{dx}.$$ 1½

(G) Solve : $2u_{x+2} + 4u_{x+1} + 8u_x = 0$. 1½

(H) Define the order of a difference equation and find the order of

$$y_{n+3} - 6y_{n+2} + 11y_{n+1} - 5y_n = \cos n.$$ 1½