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Bachelor of Science (B.Sc.) Semester—II Examination

MATHEMATICS

(Vector Calculus and Improper Integrals)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B.:—(1) Solve all the FIVE questions.
(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

- 1. (A) Find the unit tangent vector to any point on the curve $x = t^2 + 1$, y = 4t 3, $z = 2t^2 6t$. Also determine the unit tangent at the point where t = 2.
 - (B) If $\overline{A} = 2xz^2\overline{i} yz\overline{j} + 3xz^3\overline{k}$ and $\phi = x^2yz$, then find $\overline{\nabla} \times (\phi \overline{A})$ at point (1, 1, 1).

OR

- (C) Prove that $r^n \cdot \overline{r}$ is irrotational. Find the value of n when it is solenoidal.
- (D) If $\overline{F} = (2x + y)\overline{i} + (3y x)\overline{j}$, then evaluate $\int_C \overline{F} \circ d\overline{r}$ along the curve C in the xy-plane consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2).

UNIT—II

- 2. (A) Evaluate $\iint_R (x^2 + y^2) dxdy$ over the region R in the positive quadrant for which $x + y \le 1$.
 - (B) Evaluate $\int_{0}^{1} \int_{0}^{2\sqrt{x}} xy dy dx$ by changing the order of integration.

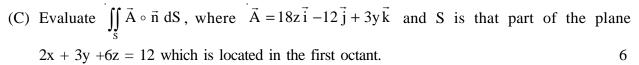
OR

- (C) Evaluate $\int_{0}^{4} \int_{y}^{4} \frac{x}{x^2 + y^2} dxdy$ by changing to polar coordinates.
- (D) Evaluate $\int_{0}^{1} \int_{y^2}^{1} \int_{0}^{1-x} x \, dz dx dy.$

UNIT—III

- 3. (A) If $\overline{F} = (2x^2 3z) \vec{i} 2xy \vec{j} 4x \vec{k}$, then evaluate $\iiint_V (\vec{\nabla} \circ \vec{F}) dV$, where V is the closed region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4.
 - (B) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by y = x and $y = x^2$.

OR



(D) Using Divergence theorem, evaluate $\iint_S \vec{F} \circ \hat{n} dS$, where $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$ and S is the surface of the solid cut off by the plane x + y + z = a from the first quadrant.

UNIT—IV

4. (A) Test the convergence of:

$$(i) \int_0^\infty \frac{x^2 + 1}{x^4 + 1} \, dx$$

(ii)
$$\int_0^\infty \frac{x}{\sqrt{x^2 + 1}} \, \mathrm{d}x \, . \tag{6}$$

(B) Test for convergence:

$$\text{(i)} \quad \int\limits_{\pi}^{4\pi} \frac{\sin \ x}{\sqrt[3]{x-\pi}} \ dx$$

(ii)
$$\int_{0}^{1} \frac{\cos x}{x^2} dx$$

OR

(C) Prove that
$$\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$
.

(D) Given that
$$\int_{0}^{\infty} \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin p\pi}, \text{ show that } |\overline{p}| \overline{1-p} = \frac{\pi}{\sin p\pi} \text{ where } 0$$

Question—V

5. (A) Find
$$\vec{\nabla} \phi$$
 if $\phi = \frac{1}{r}$, where $r = |\vec{r}|$ and $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$.

- (B) Show that if $F_1 dx + F_2 dy + F_3 dz$ is exact differential of function ϕ then $\vec{\nabla} \times \vec{F} = \vec{O}$, where $\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}$.
- (C) Evaluate $\int_{0}^{1} \int_{0}^{x} (x+2) dxdy$. 1½

(D) Evaluate
$$\int_{0}^{\pi/2} \int_{0}^{\sin \theta} r \, dr d\theta$$
. 1½

- (E) Find the area of the ellipse $x = 2 \cos \theta$, $y = 3 \sin \theta$ by using Green's theorem in the plane.
- (F) State Stoke's theorem for the surface S bounded by simple closed curve C. 1½

 $1\frac{1}{2}$

- (G) Prove that $\frac{1}{1} = 1$.
- (H) Evaluate using Beta-Gamma function $\int_{0}^{\infty} \frac{x^4}{(1+x)^9} dx$. 1½

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