Bachelor of Science (B.Sc.) Semester—II Examination

MATHEMATICS

(Vector Calculus and Improper Integrals)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the unit tangent vector to any point on the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$. Also determine the unit tangent at the point where $t = 2$. 6

(B) If $\bar{A} = 2xz^2\bar{i} - yz\bar{j} + 3xz^3\bar{k}$ and $\phi = x^2yz$, then find $\bar{\nabla} \times (\phi\bar{A})$ at point (1, 1, 1). 6

OR

(C) Prove that $r^n \cdot \bar{r}$ is irrotational. Find the value of n when it is solenoidal. 6

(D) If $\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$, then evaluate $\int_C \bar{F} \circ d\bar{r}$ along the curve C in the xy-plane consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2). 6

UNIT—II

2. (A) Evaluate $\iint_R (x^2 + y^2)\,dxdy$ over the region R in the positive quadrant for which $x + y \le 1$. 6

(B) Evaluate $\int_0^1 \int_0^{2\sqrt{x}} xy\,dydx$ by changing the order of integration. 6

OR

(C) Evaluate $\int_0^4 \int_y^4 \frac{x}{x^2 + y^2}\,dxdy$ by changing to polar coordinates. 6

(D) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dzdxdy$. 6

UNIT—III

3. (A) If $\bar{F} = (2x^2 - 3z)\vec{i} - 2xy\,\vec{j} - 4x\,\vec{k}$, then evaluate $\iiint_V (\vec{\nabla} \circ \vec{F})\,dV$, where V is the closed region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$. 6

(B) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 6

OR

(C) Evaluate $\iint_S \vec{A} \circ \vec{n}\, dS$, where $\vec{A} = 18z\,\vec{i} - 12\,\vec{j} + 3y\,\vec{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. 6

(D) Using Divergence theorem, evaluate $\iint_S \vec{F} \circ \hat{n}\, dS$, where $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ and S is the surface of the solid cut off by the plane $x + y + z = a$ from the first quadrant. 6

UNIT—IV

4. (A) Test the convergence of :

(i) $\displaystyle\int_0^{\infty} \frac{x^2 + 1}{x^4 + 1}\, dx$

(ii) $\displaystyle\int_0^{\infty} \frac{x}{\sqrt{x^2 + 1}}\, dx$. 6

(B) Test for convergence :

(i) $\displaystyle\int_{\pi}^{4\pi} \frac{\sin x}{\sqrt[3]{x - \pi}}\, dx$

(ii) $\displaystyle\int_0^{1} \frac{\cos x}{x^2}\, dx$ 6

OR

(C) Prove that $\beta(m, n) = \displaystyle\int_0^{\infty} \frac{x^{m-1}}{(1 + x)^{m+n}}\, dx$. 6

(D) Given that $\displaystyle\int_0^{\infty} \frac{x^{p-1}}{1 + x}\, dx = \frac{\pi}{\sin p\pi}$, show that $\left|\overline{p}\right.\left|\overline{1 - p}\right. = \dfrac{\pi}{\sin p\pi}$ where $0 < p < 1$. 6

Question—V

5. (A) Find $\vec{\nabla}\phi$ if $\phi = \dfrac{1}{r}$, where $r = |\vec{r}|$ and $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$. 1½

(B) Show that if $F_1 dx + F_2 dy + F_3 dz$ is exact differential of function ϕ then $\vec{\nabla} \times \vec{F} = \vec{O}$, where $\vec{F} = F_1\,\vec{i} + F_2\,\vec{j} + F_3\,\vec{k}$. 1½

(C) Evaluate $\displaystyle\int_0^{1}\int_0^{x} (x + 2)\, dxdy$. 1½

(D) Evaluate $\displaystyle\int_0^{\pi/2}\int_0^{\sin\theta} r\, dr d\theta$. 1½

(E) Find the area of the ellipse $x = 2\cos\theta$, $y = 3\sin\theta$ by using Green's theorem in the plane. 1½

(F) State Stoke's theorem for the surface S bounded by simple closed curve C. 1½

(G) Prove that $\left|\overline{1}\right. = 1$. 1½

(H) Evaluate using Beta-Gamma function $\displaystyle\int_0^{\infty} \frac{x^4}{(1 + x)^9}\, dx$. 1½