# Bachelor of Science (B.Sc.) Semester-II (C.B.S.) Examination

#### MATHEMATICS

## Compulsory Paper-II

(M.-Vector Calculus and Improper Integrals)

Time: Three Hours] [Maximum Marks: 60

- N.B. :- (1) Solve all the five questions.
  - (2) All questions carry equal marks.
  - (3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

#### UNIT-I

- (A) A particle moves along a curve whose parametric
  equations are x = e<sup>-1</sup>, y = 2 cos 3t, z = 2 sin 3 t,
  where t is time.
  - (i) Determine its velocity and acceleration at any time.
  - (ii) Find the magnitudes of the velocity and acceleration at t = 0.

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(B) Prove  $\nabla^2 r^n = n(n+1) r^{n-2}$ , where n is a constant and  $r^2 = x^2 + y^2 + z^2$ .

#### OR

- (C) Prove: (a) ∇̄x(∇̄φ) = 0, (b) ∇̄ o (∇̄x Ā) = 0
   where φ = φ(x, y, z) and Ā=A₁ī+A₂j̄+A₃k̄ are differentiable scalar point function and vector point function of x, y, z.
  - (D) If  $\overline{A} = (2y + 3) \overline{1} + x z \overline{1} + (yz x) \overline{k}$ , then evaluate  $\int_C \overline{A} o d\overline{r}$  along the path  $C : x = 2t^2$ , y = t,  $z = t^3$  from t = 0 to t = 1.

#### UNIT-II

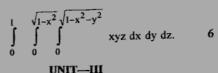
- 2. (A) Evaluate  $\iint \frac{xy}{\sqrt{1-y^2}} dxdy$  over the positive quadrant of the circle  $x^2 + y^2 = 1$ .
  - (B) Change the order of integration:



OR

2. (C) Evaluate  $\int_{0}^{a} \int_{y}^{a} \frac{x \, dy dx}{x^2 + y^2}$  by changing to polar coordinates.

(D) Evaluate :



- 3. (A) Evaluate  $\iiint_V (2x + y) dv$ , where v is the closed region bounded by the cylinder  $z = 4 x^2$  and the planes x = 0, y = 0, y = 2 and z = 0.
  - (B) Evaluate  $\oint_C (x^2 + y^2) dx + 3xy^2 dy$  by using Green's theorem in a plane, where C is a circle of radius 2 with centre at the origin of the xy-plane, traversed in the positive sense.

## OR

theorem, where  $\vec{l} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$  and S is the surface of the region bounded by x = 0, y = 0, y = 3, z = 0 and x + 2z = 6.

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(Coptd )

(D) Verify Stoke's theorem for :

 $\vec{A} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ , where S is the upper half surface of the sphere  $x^2 + y^2 + z^2 = 1$  and C is its boundary.

#### UNIT-IV

4. (A) (i) Test the convergence of:

$$\int_0^\infty \frac{x^2+1}{x^4+1} dx$$

- (ii) Prove that an absolutely convergent integral converges.
- (B) Define Gamma function of n. Prove that  $\sqrt{(n+1)} = n \sqrt{n}$ , n > 0 and  $\sqrt{(n+1)} = n$ !, n = 1, 2, 3...

### OR

- 4. (C) Prove the relation  $\beta$  (m, n) =  $\frac{\boxed{m} \boxed{N}}{\boxed{(m+n)}}$ , m, n > 0.
  - (D) Prove that  $\ln = \int_{0}^{1} \left(\log \frac{1}{x}\right)^{n-1} dx$ . Hence evaluate

$$\int_{0}^{1} \sqrt{\log \frac{1}{x}} dx$$

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(Contd.)

- (A) Find the value of n for which F = (n-4) xi + yj + zk is solenoidal.
  - (B) Find the work done by a force field F = xi y²j in moving a particle from (1, 1) to (2, 2) along a straight line y = x.
  - (C) Find the area of the rectangular region R bounded by the lines x = 0, x = 2, y = 0 and y = 3, using double integral.
  - (D) Evaluate  $\int_{0}^{1} \int_{0}^{2} (x+3) dx dy$ . 1½
  - (E) Apply Green's theorem to prove that the area enclosed by a simple plane curve C is

$$\frac{1}{2} \oint_C (x \, dy - y \, dx).$$
 1½

(F) Show that:

$$\iint_{S} (ax \bar{i} + by \bar{j} + cz \bar{k}) \circ \hat{n} dS = \frac{4}{3}\pi (a+b+c) \text{ where S}$$
is the surface of the sphere  $x^2 + y^2 + z^2 = 1$ , by divergence theorem.

(G) Test the convergence of:

$$\int_{-\frac{1}{x^{3}+5}}^{\frac{1}{x^{3}+5}} by comparison test.$$

(11) Evaluate 
$$\int_{0}^{x} \frac{x^{8}(1-x^{8})}{(1+x)^{24}} dx$$

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