NKT/KS/17/5108

Bachelor of Science (B.Sc.) Semester—III (C.B.S.) Examination

MATHEMATICS (M_5-Advanced Calculus, Sequence and Series)

Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) If two functions f(x) and F(x) are continuous in [a, b] and derivable in (a, b) and $F'(x) \neq 0$ for any value of x in [a, b], then prove that there exist at least one value $c \in (a, b)$ such that :

$$\frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(c)}{F'(c)}$$ 6

(B) In the Cauchy's mean value theorem :

(i) If $f(x) = \sqrt{x}$, $g(x) = \frac{1}{\sqrt{x}}$, $x \in [a, b]$ then show that the value $c \in (a, b)$ is the geometric mean between a and b, where a, b, > 0.

(ii) If $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$, $x \in [a, b]$, then show that the value $c \in (a, b)$ is the harmonic mean between a and b, where a, b > 0. 6

OR

(C) Let f(x, y) and g(x, y) be defined in the open region $D C R^2$. If f(x, y) and g(x, y) both are continuous at $P_o(x_o, y_o) \in D$, then prove that f(x, y) – g(x, y) is also continuous at $P_o(x_o, y_o)$. 6

(D) Expand $f(x, y) = e^x \cos y$ by Taylor's series in powers of x and y such that it include all terms upto third degree. 6

UNIT—II

2. (A) Find the envelope of the family of lines $x \cos \alpha + y \sin \alpha = \ell \sin \alpha \cos \alpha$, where α is a parameter. Also give the geometrical interpretation. 6

(B) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ when $a^m b^n = c^{m+n}$, where a and b are parameters and c is a constant. 6

OR

(C) Discuss the maximum and minimum values of $u = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. 6

(D) Use Lagrange's multiplier method to find the maximum and minimum values of $u = x + y + z$ subject to the condition $x^2 + y^2 + z^2 = \ell$. 6

UNIT—III

3. (A) If the sequences $<y_n>$ and $<z_n>$ converge to ℓ and if $y_n < x_n < z_n \ \forall \ n \in N$, then prove that the sequence $<x_n>$ also converges to l. 6

(B) Show that the sequence $\left\langle \frac{n}{n+1} \right\rangle, \forall \ n \in N$, is monotonic increasing, bounded and converges to 1. 6

OR

(C) Prove that the sequence $<x_n>$ converges if and only if it is a Cauchy sequence. 6

(D) Prove that the sequence $\left\langle \frac{e^n}{n} \right\rangle$ is monotonic increasing, bounded below but not bounded above. 6

UNIT—IV

4. (A) Test the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{\left(n+\sqrt{n}\right)^n}{2^n \ n^{n+1}}$$ by root test. 6

(B) Examine the convergence of the series :

$$\frac{x^3}{1.3}+\frac{x^4}{2.4}+\frac{x^6}{3.5}+\ldots\frac{x^{2n}}{n(n+2)}+\ldots$$ by ratio test. 6

OR

(C) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is convergent if $p > 1$ and divergent if $0 < p \le 1$. 6

(D) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$ is conditionally convergent. 6

Question—5

5. (A) Verify Rolle's theorem for $f(x) = x^2$ in $[-1, 1]$. 1½

(B) Using $\in$-δ definition, show that :

$$\lim_{(x, y) \to (1, 2)} (3x + y) = 5.$$ 1½

(C) Find the envelope of $y = mx + \frac{1}{m}$, where m is a parameter. 1½

(D) Define extreme point and saddle point of a function f(x, y). 1½

(E) Prove that $\lim_{n \to \infty} \frac{2+3\times10^n}{1+5\times10^n} = \frac{3}{5}$. 1½

(F) Evaluate $\lim_{n \to \infty} \left(\sqrt{n+1} - \sqrt{n}\right)$. 1½

(G) Test the convergence of the series whose n^{th} term is $\frac{n!}{n^n}$. 1½

(H) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent by integral test. 1½