Bachelor of Science (B.Sc.) Semester—IV Examination

MATHEMATICS (PARTIAL DIFFERENTIAL EQUATION AND CALCULUS OF VARIATION)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}.$$ 6

(B) Prove a necessary and sufficient condition that there exists between two functions u(x, y) and v(x, y) a relation F(u, v) = 0, not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$. 6

OR

(C) Verify the equation :

$$2xzdx + 2yzdy - (x^2 + y^2)(z - 1)\,dz = 0,$$

is integrable and if it is so, then solve it. 6

(D) Eliminate the arbitrary function f from the equation $z = f\left(\frac{xy}{z}+z\right)$. 6

UNIT—II

2. (A) Find the general solution of the partial differential equation :

$$(y + zx)\,p - (x + yz)q = x^2 - y^2.$$ 6

(B) Show that the equations :

$$xp = yq\ ,\ z(xp + yq) = 2xy,$$

are compatible and find their solution. 6

OR

(C) Find the complete integral of the partial differential equation :

$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$$

by using Charpit's method. 6

(D) Show that a complete integral of

$$f\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right) = 0, \text{ is } u = ax + by + \phi(a, b)\,z + c,$$

where a, b, c are arbitrary constants and f(a, b, ϕ) = 0. Further find also the complete integral of $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}$. 6

UNIT—III

3. (A) Solve $(D^3 - 4D^2D' + 4DD'^2)z = \sin(y + 2x)$ where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 6

(B) Solve $(D^2 - DD' + D' - 1)z = \cos(x - 2y) + x^2$ where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 6

OR

(C) Solve $(D^2 - D')z = 2y - x^2$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 6

(D) Solve $(x^2D^2 + y^2D'^2 + 2xyDD')z = x^ny^m$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$, by using $x = e^u$ and $y = e^v$. 6

UNIT—IV

4. (A) Find the extremal of the functional $I[y(x)] = \int_0^1 [xy' - y'^2]\,dx$, $y(0) = 1$, $y(1) = \frac{1}{4}$. 6

(B) Find the extremal of the functional :

$$I[y(x), z(x)] = \int_1^2 [z'^2 - xy'z]\,dx,\ y(1) = z(1) = 1,\ y(2) = -\frac{1}{6},\ z(2) = \frac{1}{2}.$$ 6

OR

(C) Among the plane smooth curves joining the points $A(x_0, y_0)$ and $B(x_1, y_1)$ find that one which generates the surface with the least area upon rotation around the OX-axis. 6

(D) Find Euler-Ostrogradsky's equation for the functional :

$$I[u(x, y, z)] = \iiint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + 2uf\right] dxdydz.$$ 6

Question-V

5. (A) Solve the equation $zydx + zxdy + 2xydz = 0$. 1½

(B) Form a partial differential equation by eliminating arbitrary constants a and b from the equation $ax^2 + by^2 - z^2 = 1$. 1½

(C) Solve $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$. 1½

(D) Show that the equations $p = P(x, y)$ and $q = Q(x, y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. 1½

(E) Solve $xys = 1$, where $s = \frac{\partial^2 z}{\partial x \partial y}$. 1½

(F) Find particular integral of $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$. 1½

(G) Let $I[y(x)] = \int_0^1 [y(x)]^2\,dx$, be a functional. If $y(x) = \sqrt{1 + x^2}$ then find $I[y(x)]$. 1½

(H) If F is linearly dependent on y' defined by $F = P(x, y) + y'Q(x, y)$, then from Euler's equation, show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. 1½