

Bachelor of Science (B.Sc.) Semester—V
(C.B.S.) Examination
MATHEMATICS

Paper—I
(M₉-Analysis)

Time—Three Hours]

[Maximum Marks—60

- N.B. :—** (1) Solve all the **FIVE** questions.
 (2) All questions carry equal marks.
 (3) Question Nos. **1** to **4** have an alternative.
 Solve each question in full or its alternative
 in full.

UNIT—I

1. (A) Show that the Fourier Series for the periodic function defined by $f(x) = 0$, $-\pi \leq x < 0$ and $f(x) = x^2$, $0 \leq x < \pi$ is :

$$f(x) = \frac{\pi^2}{6} + 2 \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2} + \pi \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)^3}$$

6

- (B) Let $f(x)$ be an integrable function defined on the interval $-\pi \leq x \leq \pi$. If $f(x)$ is odd, then prove that its Fourier Series has only sine terms and the coefficients are given by :

$$a_n = 0, n = 0, 1, 2, 3, \dots$$

$$\text{and } b_n = \frac{2}{\pi} \int_0^\pi f(x) \sin nx \, dx, n = 1, 2, 3, \dots$$

6

OR

- (C) Show that the cosine series for x^2 is :

$$x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}; \quad -\pi \leq x \leq \pi \quad 6$$

- (D) Expand $f(x)$ in a Fourier Series on the interval $-2 \leq x < 2$ if $f(x) = 0$ for $-2 \leq x < 0$ and $f(x) = 1$ for $0 \leq x < 2$. 6

UNIT—II

2. (A) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P such that :

$$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon. \quad 6$$

- (B) Let $f \in \mathcal{R}$ on $[a, b]$. For $a \leq x \leq b$, put $F(x) = \int_a^x f(t) \, dt$, where F is continuous on $[a, b]$.

If f is continuous at a point x_0 of $[a, b]$, then prove that F is differentiable at x_0 and $F'(x_0) = f(x_0)$. 6

OR

Contd.

(C) If $F \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove that :

(i) $\int_a^b f g \, d\alpha \in R(\alpha)$

(ii) $|f| \in R(\alpha)$ and $\left| \int_a^b f \, d\alpha \right| \leq \int_a^b |f| \, d\alpha$. 6

(D) Suppose F and G are differentiable functions on $[a, b]$, $F' = f \in R$ and $G' = g \in R$, then prove that :

$$\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx.$$

6

UNIT—III

3. (A) If $f(z) = u + iv$ is an analytic function in a domain D , prove that the curves $u = \text{constant}$, $v = \text{constant}$ form two orthogonal families. 6

(B) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that $u + iv$ is an analytic function of $x + iy$. 6

OR

(C) If $f(z)$ is an analytic function with constant modulus, then prove that it is constant. 6

(D) If $u = x^2 - y^2$, $v = \frac{y}{(x^2 + y^2)}$, then show that both u and v satisfy Laplace's equation, but $u + iv$ is not an analytic function of z . 6

UNIT—IV

4. (A) Let a rectangular domain R be bounded by $x = 0$, $y = 0$, $x = 2$, $y = 1$ in z -plane. Determine the region R' of w -plane into which R is mapped under the transformation $w = z + (1 - 2i)$. 6

(B) Prove that the cross ratio remains invariant under a Bilinear Transformation. 6

OR

(C) Find the bilinear transformation which maps the points $z = -2, 0, 2$ into the point $w = 0, i, -i$ respectively. 6

(D) Establish the relation $w = \frac{iz + 2}{4z + i}$ transforms the real axis in z -plane to a circle in the w -plane. Find the centre and the radius of the circle and the point in the z -plane which is mapped on the centre of the circle. 6

5. (A) If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, then prove

$$\text{that } a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx; \quad n = 1, 2, 3, \dots$$

1½

(B) Find the Fourier coefficient b_n for the function $f(x) = x, -\pi \leq x \leq \pi$ 1½

(C) For any partition P of $[a, b]$, prove that $L(P, f) \leq U(P, f)$. 1½

Contd.

(D) If $f \in R(\alpha)$ on $[a, b]$, then prove that $cf \in R(\alpha)$ and

$$\int_a^b cf \, d\alpha = c \int_a^b f \, d\alpha \text{ for every constant } c. \quad 1\frac{1}{2}$$

(E) If $w = f(z) = u + iv$ be an analytic function in a

domain D , then prove that $\frac{dw}{dz} = \frac{\partial w}{\partial z}$. 1½

(F) Prove that $u = y^3 - 3x^2y$ is a harmonic function.

1½

(G) Find the fixed points of the bilinear transformation

$$w = \frac{z-1}{z+1}. \quad 1\frac{1}{2}$$

(H) Show that $w = iz + i$ maps half plane $x > 0$ onto half plane $v > 1$. 1½