

(D) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 + a \cos \alpha}, a^2 < 1. \quad 6$$

UNIT—V5. (A) Prove that a finite point set has no limit point. $1\frac{1}{2}$ (B) If X is a metric space and $E \subset X$, then prove that $\bar{E} = \text{closure of } E = E \cup E'$ is closed where $E' \equiv$ the set of all limit points of E in X . $1\frac{1}{2}$ (C) Prove that if F is closed and K is compact, then $F \cap K$ is compact. $1\frac{1}{2}$ (D) If $\{K_n\}$ is a sequence of non-empty compact sets such that $K_n \supset K_{n+1}$ ($n = 1, 2, 3, \dots$), then prove

$$\text{that } \bigcap_{n=1}^{\infty} K_n \text{ is not empty.} \quad 1\frac{1}{2}$$

(E) Let R be a ring. Prove that if $a, b \in R$, then $(a + b)^2 = a^2 + ab + ba + b^2$. $1\frac{1}{2}$ (F) If R is a ring and $a \in R$, let $r(a) = \{x \in R / ax = 0\}$. Prove that $r(a)$ is a right-ideal of R . $1\frac{1}{2}$ (G) Evaluate $\int_0^{1+i} z \, dz$ along the line $z = 0$ to $z = 1 + i$. $1\frac{1}{2}$ (H) If $f(z) = \frac{1}{(z-5)^3(z-4)^2}$ then find the poles with order. $1\frac{1}{2}$

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NTK/KW/15/5884**Bachelor of Science (B.Sc.) Semester—V (C.B.S.)****Examination****MATHEMATICS****(M₁₀-Metric Space, Complex Integration and Algebra)****Paper—II**

Time—Three Hours]

[Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Solve each question in full or its alternative in full.

UNIT—I1. (A) Prove that every infinite subset of a countable set A is countable. 6 (B) Let X be an infinite set. For $p \in X$ and $q \in x$, define

$$d(p, q) = \begin{cases} 1 & (\text{if } p \neq q) \\ 0 & (\text{in } p = q) \end{cases}$$

Prove that this d is a metric. 6 **OR**

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Contd.

- (C) Let $\{E_\alpha\}$ be a (finite or infinite) collection of sets E_α . Then prove that

$$\left(\bigcup_{\alpha} E_{\alpha}\right)^c = \bigcap_{\alpha} (E_{\alpha}^c) \quad 6$$

- (D) For any collection $\{G_\alpha\}$ of open sets, prove that $\bigcup_{\alpha} G_\alpha$ is open. Hence, for any collection $\{F_\alpha\}$ of closed sets, prove that $\bigcap_{\alpha} F_\alpha$ is closed. 6

UNIT—II

2. (A) Prove that closed subsets of compact sets are compact. 6
- (B) If $\{I_n\}$ is a sequence of intervals in \mathbb{R}^1 , such that $I_n \supset I_{n+1}$ ($n = 1, 2, 3, \dots$), then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty. 6

OR

- (C) Let Y be a subspace of a complete metric space X . then prove that Y is complete if and only if Y is closed. 6
- (D) Let a subset E of the real line \mathbb{R}^1 be connected and $x, y \in E$ such that $x < z < y$, then prove that $z \in E$. 6

UNIT—III

3. (A) Prove that a finite integral domain is a field. 6

- (B) If ϕ is a homomorphism of a ring R into a ring R' with kernel $I(\phi)$, then prove that

- (a) $I(\phi)$ is a subgroup of R under addition and
 (b) if $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$. 6

OR

- (C) Prove that the homomorphism ϕ of R into R' is an isomorphism if and only if $I(\phi) = \text{kernel of } \phi = \{0\}$. 6
- (D) If \cup is an ideal of R , let $r(\cup) = \{x \in R / xu = 0 \text{ for all } u \in \cup\}$. Prove that $r(\cup)$ is an ideal of R . 6

UNIT—IV

4. (A) If $f(z)$ is analytic within and on a closed contour C , and if a is any point within C , then prove that $f(a) = \frac{1}{2\pi i} \int_C \frac{f(z) dz}{(z-a)}$. 6

- (B) Find the value of the integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight line from $z = 0$ to $z = 1 + i$. 6

OR

- (C) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3, and infinity and show that their sum is zero. 6

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