NRJ/KW/17/3136

Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination
METRIC SPACE, COMPLEX INTEGRATION AND ALGEBRA
Paper—2
(Mathematics)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all *five* questions.
(2) All questions carry equal marks.
(3) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full.

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UNIT—I

1. (A) Define uncountable set. If A is the set of all sequences whose elements are the digits 0 and 1, then prove that A is uncountable. 6

(B) Let X be a non-empty set and d be a real function of ordered pairs of elements of X which satisfies the conditions :

(i) $d(x, y) = 0$ iff $x = y$

(ii) $d(x, y) \leq d(x, z) + d(z, y)$.

Then prove that d is a metric on X. 6

OR

(C) Let X be a metric space and $E \subset X$. Prove that E is open if and only if its complement is closed. 6

(D) Let X be a metric space and $E \subset X$. If $\bar{E}$ denotes the closure of E, then prove that :

(i) $\bar{E}$ is closed

(ii) $E = \bar{E}$ iff E is closed. 6

UNIT—II

2. (A) Prove that in a metric space, closed subsets of compact sets are compact. 6

(B) If E is an infinite subset of a compact set K, then prove that E has a limit point in K. 6

OR

(C) Let Y be a subspace of a complete metric space X. Prove that Y is complete if and only if Y is closed. 6

(D) Define connected set. If E is a connected set, then find whether closure of E and interior of E are always connected. 6

UNIT—III

3. (A) Show that the commutative ring D is an integral domain if and only if for a, b, c $\in$ D with $a \neq 0$ the relation $ab = ac$ implies that $b = c$. 6

(B) If U, V are ideals of ring R and $U + V = \{u + v/u \in U, v \in V\}$, then prove that U + V is also an ideal. 6

OR

(C) If U is an ideal of the ring R, then prove that the quotient ring R/U is a homomorphic image of R. Also prove that kernel of homomorphism is an ideal U. 6

(D) Let R and R' be rings and ϕ be a homomorphism of R into R'. Prove that ϕ is an isomorphism if and only if $I(\phi) = (0)$. 6

(Contd.)

UNIT—IV

4\. (A) Find the value of the integral $\int_{2-i}^{2+i}(2x+iy+2)dz$ along the straight line joining the points (1–i) and (2+i). 6

(B) Verify Cauchy's integral theorem for the function $f(z) = e^z$ along the boundary of the triangle with vertices at the points $1 + i$, $-1 + i$ and $-1 - i$. 6

OR

(C) State and prove Cauchy's Residue theorem for analytic function. 6

(D) If a function f(z) is analytic except at finite number of singularities (including that at infinity), then prove that the sum of residues of these signularities is zero. Hence show that the residue of $\frac{z^3}{(z-1)(z-2)(z-3)}$ at $z = \infty$ is -6. 6

QUESTION—V

5\. (A) Show that a finite point subset of a metric space has no limit points. 1½

(B) If A and B are subsets of a metric space X, then prove that $A \subset B \Rightarrow A' \subset B'$. 1½

(C) If F is closed and K is compact, then prove that $F \cap K$ is compact. 1½

(D) Define separated set and connected set. 1½

(E) If R is a ring and $a, b \in R$, then show that $(a + b)^2 = a^2 + ab + ba + b^2$. 1½

(F) If U is an ideal of ring R with unity 1 and $1 \in U$, then prove that $U = R$. 1½

(G) Prove that $\int_C \frac{dz}{z-a} = 2\pi i$, if C is a circle $|z - a| = r$. 1½

(H) Find the poles of $f(z) = \frac{z+1}{z^2(z-3)}$. Also state which is a simple pole. 1½