NJR/KS/18/6813

Bachelor of Arts (B.A.) Part–I First Semester Examination

MATHEMATICS (Algebra and Trigonometry)

Optional Paper–1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **five** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Reduce the matrix A to its normal form and find its rank where :

$$A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 1 \end{bmatrix}$$ 6

(B) Investigate for what values of λ and μ, the simultaneous equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$ have

(i) no solution

(ii) a unique solution,

(iii) an infinite number of solutions. 6

OR

(C) Find the eigenvalues of the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$$

and find the corresponding eigen vector of the matrix associated with only one eigen value. 6

(D) Show that the matrix A satisfies Cayley-Hamilton's theorem where

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}.$$ 6

UNIT–II

2. (A) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in arithmetical progression and hence solve $x^3 - 12x^2 + 39x = 28$. 6

(B) Solve the reciprocal equation :

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$$ 6

OR

(C) Solve the Cubic equation $x^3 - 6x - 9 = 0$ by Cardon's method. 6

(D) Solve the equation

$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$ by Ferrari's method. 6

UNIT–III

3. (A) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$, then prove that x_1, x_2, x_3 ..... ad inf $= -1$. 6

(B) Solve the equation $x^5 \;\; 1 = 0$, using DeMoivre's theorem. 6

OR

(C) If $\tan(\theta + i\phi) = \tan\alpha + i \sec\alpha$, then prove that $e^{2\phi} = \pm \cot\frac{\alpha}{2}$. 6

(D) If $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$. Then prove the Gregory series

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - ....$$ 6

UNIT–IV

4. (A) Prove that the set $G = \{1, w, w^2\}$ with $w^3 = 1$ is an abelian group of order 3 with respect to multiplication. 6

(B) In a group (G, 0) prove that :

(i) identity element in G is unique;

(ii) inverse of every element in G is unique;

(iii) $(a^{-1})^{-1} = a, \ \forall \ a \in G$. 6

OR

(C) Prove the necessary and sufficient condition that a non-empty subset H of group (G, o) is a subgroup of G if and only if :

(i) $a, b \in H \Rightarrow a \circ b^{-1} \in H$ and

(ii) $a \in H \Rightarrow a^{-1} \in H$. 6

(D) Prove that every permutation can be expressed as a product of transpositions. Express $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ as a product of disjoint cycles and find whether permutation f is even or odd. 6

QUESTION–V

5. (A) Write the augmented matrix for the system of equations $x + 2y + 3z = 4$, $2x + 2y + 8z = 7$, $x - y + 9z = 1$. 1½

(B) Find the characteristic equation for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 1½

(C) Form the equation whose roots are 1, 2 and 3. 1½

(D) If α, β, γ are the roots of cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2$. 1½

(E) Find all the values of $(-1)^{1/3}$. 1½

(F) Show that : $\log(xi) = \log x + i\left(2n + \frac{1}{2}\right)\pi$. 1½

(G) Let G = {0, 1, 2, 3, 4} be a finite abelian group of order 5 with respect to addition modulo 5. Find inverses of 1, 2, 3. 1½

(H) Find the cycles and the orbits of the permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 2 & 1 & 6 & 5 & 7 \end{pmatrix}.$$ 1½