NRT/KS/19/5755

Bachelor of Arts (B.A.) First Semester Examination

MATHEMATICS (Algebra and Trigonometry)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find rank of the matrix by row-reduction :

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 2 \\ 3 & -2 & 2 & 4 \end{bmatrix}$$ 6

(B) Solve the equations :

$x + y + z = 3;\ x + 2y + 3z = 4;$

$x + 4y - 9z = 6$ 6

OR

(C) Find eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 3 & 2 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$ 6

(D) Verify Caley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and hence find A^{-1}. 6

UNIT—II

2. (A) Solve the equation :

$x^3 - 6x^2 + 3x + 10 = 0$, if roots are in arithmetic progression. 6

(B) Solve the equation $x^4 + 2x^2 - 22x + 7 = 0$, if one of the roots is $2+\sqrt{3}$. 6

OR

(C) Solve by Cardon's method :

$x^3 + x^2 - 16x + 20 = 0.$ 6

(D) Solve by Ferrari's method :

$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0.$ 6

UNIT—III

3. (A) Find all the values of $(32)^{\frac{1}{6}}$. 6

(B) Expand $\cos 7\theta$ in terms of $\cos\theta$ and $\sin\theta$. 6

OR

(C) Prove that :

(i) $\cosh^2 x - \sinh^2 x = 1$

(ii) $\tanh^{-1} x = \sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ 6

(D) Separate $\log_e(x + iy)$ into real and imaginary parts. 6

UNIT—IV

4. (A) Prove that the set of fourth roots of unity form an abelian group under multiplication. 6

(B) If (G, o) is a group and a, b $\in$ G, then prove that equations $a \circ x = b$ and $y \circ a = b$ have unique solutions in G. 6

OR

(C) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 2 & 4 & 1 & 3 \\ 3 & 1 & 2 & 4 \end{pmatrix}$ then find $f \circ g$, $g \circ f$, $(f \circ g)^{-1}$, $(g \circ f)^{-1}$. Is $f \circ g = g \circ f$? 6

(D) Show that any two right (or left) cosets of subgroup are either disjoint or identical. 6

Question—V

5. (A) Give an example of echelon form of a matrix which is not in a normal form. 1½

(B) Show that A and A^T have same eigen values. 1½

(C) Form an equation whose one of the roots is $1 + 2i$. 1½

(D) Find an equation whose roots are reciprocal of the roots of $x^3 - 2x^2 + 3x + 7 = 0$. 1½

(E) Prove that :

(i) $\cos iz = \cos hz$

(ii) $\sin iz = i \sin hz$. 1½

(F) Prove that $\log i = \frac{\pi}{2} i$. 1½

(G) Prove that in a group G, $(a^{-1})^{-1} = a \ \forall \ a \in G$. 1½

(H) Define a subgroup of a group. 1½