NKT/KS/17/9377

Bachelor of Arts (B.A.) Part—I Second Semester Examination

STATISTICS

(Probability Distribution)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

Note :—All questions are compulsory and carry equal marks.

1. (A) Obtain the mode of Poisson distribution when its parameter λ is not an integer and when it is an integer.

   (B) Derive the mgf of Poisson distribution. Hence find its mean, variance and third central moment. 5+5

OR

   (E) Derive the pmf of binomial distribution. Find its first three raw moments about origin. Hence find μ_2, μ_3 and β_1. Comment on the skewness of binomial distribution. 10

2. (A) Derive the pmf of geometric distribution. Find its mgf. Hence find its mean and variance. State and prove the lack of memory property of geometric distribution. If the probability of having a male child is 0.5, then find the probability that :

   (i) a family's 4^{h} child is their first son.

   (ii) a family's 3rd child is their second son. 10

OR

   (E) State the pmf of negative binomial distribution. Derive its mgf and hence find its mean.The probability is 0.30 that a child exposed to a certain contagious disease will catch it. What is the probability that the 10^{th} child exposed to the disease will be the 3rd to catch it ?

   (F) State the pmf of hypergeometric distribution. Find its mean. 5+5

3. (A) State the chief characteristics of normal distribution and normal density curve.

   (B) Obtain the mgf of normal distribution.

   (C) A r.v. X follows continuous uniform distribution over an interval [a, b]. Find the mean and variance of X.

   (D) Show that an odd ordered central moments of normal distribution are zero. 2½×4=10

OR

   (E) Find the mean, mode and median of normal distribution. Show that a linear combination of K independent normally distributed variables also follows normal distribution. 10

4. (A) State the pdf of gamma distribution with one parameter. Obtain its mgf. Hence find μ_2, μ_3 & μ_4. Calculate β_1, & β_2. Comment on the skewness and kurtosis of this distribution. 10

OR

(E) State the pdf of exponential distribution. Find its mgf. State and prove the lack of memory property of this distribution.

(F) State the pdf of beta distribution of first type. Obtain the r^{th} moment about origin. Hence find the mean and variance of this distribution. 5+5

5. Solve any **TEN** of the following questions :—

(A) Find the mean and variance of Bernoulli distribution.

(B) If the mgf of a probability distribution is $\left(\frac{1+4e^t}{5}\right)^{12}$, then identify its distribution. Hence state its mean and variance.

(C) If for a Poisson distribution, P[X = 1] = P[X = 2], find the parameter λ.

(D) Define a standard normal variable. State its pdf.

(E) If X_1 and X_2 are independently distributed normal variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively, state the distribution of $X_1 + 2X_2$.

(F) State one real life example of a r. v. where discrete uniform distribution is applicable

(G) Give one example of a r.v. following geometric distribution.

(H) If the mean and variance of geometric distribution are 4 and 5 respectively, find its parameter 'p'.

(I) A committee of 3 persons is to be randomly chosen out of 7 women and 8 men in a department. Let X denote number of women in the chosen committee. What is the probability distribution of X ? State its parameters.

(J) If the pdf of exponential distribution is

$$f(x) = \alpha e^{-\alpha x} \quad , \quad 0 < x < \infty$$
$$= 0 \quad , \quad \text{otherwise}$$

then state its mean.

(K) Write the pdf of gamma distribution with two parameters.

(L) State a continuous probability distribution that has mean = variance. 1×10=10