NIR/KW/18/5852

Bachelor of Arts (B.A.) Third Semester Examination

MATHEMATICS

(Advanced Calculus, Sequence and Series)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or it alternative in full.

UNIT—I

1. (A) If f(x, y) and g(x, y) are two functions then prove that limit of a product of these two functions equals the product of their limits. 6

(B) Prove that the function $f(x, y) = xy$ is continuous everywhere by using $\in - \delta$ definition of continuity. 6

OR

(C) Expand $f(x, y) = x^2 + xy + y^2$ in powers of $(x - 2)$ and $(y - 3)$. 6

(D) Verify Cauchy's mean value theorem for the function x^2 and x^4 in the interval [a, b], a, b being positive. 6

UNIT—II

2. (A) Find envelope of the straight line $x/a + y/b = 1$ when $ab = c^2$ and c a is constant. 6

(B) Show that minimum value of $u = xy + \left(\frac{a^3}{x}\right) + \left(a^3/y\right)$ is $3a^2$. 6

OR

(C) Determine the maxima and minima of $x^2 + y^2 + z^2$ when $ax^2 + by^2 + cz^2 = 1$. 6

(D) Show that if the perimeter of a triangle is constant, its area is maximum when it is equilateral. 6

UNIT—III

3. (A) Prove that every convergent sequence is bounded. Give an example to show that converse is not true. 6

(B) Show that the sequence $<x_n>$ where $x_n = 1 + 1/3 + 1/3^2 + ... + 1/3^n$ is a convergent sequence and converges to 3/2. 6

OR

(C) Show by applying Cauchy's convergence criterion that the sequence $<x_n>$ given by :

$x_n = 1 + 1/3 + 1/5 + ... + \frac{1}{2n-1}$ diverges. 6

(D) If $<x_n>$ and $<y_n>$ be two sequences such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$ where x and y are finite numbers then prove that $\lim_{n\to\infty}\left(x_n/y_n\right) = x/y$ provided $y \neq 0$. 6

UNIT—IV

4. (A) Test the convergence of the following series using comparison test, whose n^{th} term is given as :

(i) $u_n = \left[\frac{1}{n} - \log \left(\frac{n+1}{n} \right) \right]$

(ii) $u_n = \frac{n+1}{(n+2)^2}$. 6

(B) Prove that the nature of an infinite series remains unaltered by the addition or removal of a finite number of terms. 6

OR

(C) Prove that the series $\sum (-1)^n \frac{x^n}{n}$ is :

(i) Convergent if $|x| < 1$

(ii) Divergent if $|x| > 1$. 6

(D) Prove that the series $\sum (-1)^n \sin 1/n$ is conditionally convergent. 6

Question—V

5. (a) Verify Rolle's theorem for $f(x) = x^2$ in $[-1, 1]$. 1½

(b) Show that the function :

$$f(x, y) = x^2 + 2y \text{ when } (x, y) \neq (1, 2)$$
$$= 0 \text{ when } (x, y) = (1, 2)$$

has a removable discontinuity at (1, 2). 1½

(c) Find the envelope of the family of straight lines :

$y = mx + 1/m$. 1½

(d) Define Maxima and Minima of function of two variables. 1½

(e) Define Limit of a sequence. 1½

(f) Prove that a monotonic decreasing sequence is convergent if and only if it is bounded. 1½

(g) Prove that an infinite series $u_1 + u_2 + \ldots + u_n + \ldots$ is convergent if $\lim_{n \to \infty} (u_n)^{1/n} < 1$. 1½

(h) Prove that an absolute convergent series is convergent. 1½