NRT/KS/19/5854

Bachelor of Arts (B.A.) Third Semester Examination

STATISTICS

(Statistical Methods)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) Define (i) joint p.d.f. (ii) marginal p.d.f. (iii) conditional p.d.f. (iv) conditional mean and (v) conditional variance of a continuous bivariate probability distribution. The p.d.f. of a continuous bivariate distribution is :

$$f(X, Y) = \begin{cases} X+Y, & 0<X<1 \\ & 0<Y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find :

(i) Marginal p.d.fs of X and Y.

(ii) Conditional p.d.f. of Y given X = x.

(iii) Conditional mean of Y given $X = \frac{1}{2}$.

(iv) Conditional variance of Y given $X = \frac{1}{2}$. 10

OR

(E) Define :

(i) Bivariate p.d.f. of random variables X and Y.

(ii) Conditional p.d.f. of X given Y = y.

(iii) Conditional mean of X given Y = y.

(iv) Conditional variance.

(v) Stochastic independence of rv's X and Y. If the random variables X and Y have joint p.d.f.

$$f(X, Y) = \begin{cases} 12XY(1-Y), & 0<X<1. \\ & 0<Y<1. \\ 0, & \text{elsewhere} \end{cases}$$

Check whether X and Y are Stochastically independent. 10

2. (A) State the p.d.f. of Bivariate normal distribution of r.v. (X, Y). Find its m.g.f. and hence find means of X and Y. Let X and Y have a bivariate normal distribution with means μ_1 and μ_2, positive variances σ_1^2 and σ_2^2 and correlation coefficient. Then using m.g.f. show that X and Y are independent iff $\rho = 0$. 10

OR

(E) State the p.m.f. of trinomial distribution. Find the marginal p.m.f. of X. A manufactured item is classified as 'good', a 'second' or 'defective' with probabilities $\frac{6}{10}, \frac{3}{10}$ and $\frac{1}{10}$ respectively. Fifteen such items are selected at random from the production line. Let X denote the number of good items. Y denote the number of seconds and 15-X-Y the number of defectives.

(a) Give the joint p.m.f. of X and Y.

(b) Find $P(X = 10, Y = 4)$.

(c) Find $P(X \le 1)$. 10

3. (A) Let $X_1, X_2 \ldots\ldots X_n$ be a random sample of size n from exponential distribution. Find the probability distribution of $\sum_{i=1}^{n} X_i$.

(B) If the joint of random variables X_1 and X_2 is :

$$f(X_1, X_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1 > 0, x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find :

(a) the joint p.d.f. of r.v.s $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$.

(b) the marginal p.d.f. of Y_2. 5+5

OR

(E) Suppose that random variables X and Y have joint density function given by :

$$f(X, Y) = \begin{cases} C(2X+Y), & 2 < X < 6, 0 < Y < 5 \\ 0 & \text{elsewhere} \end{cases}$$

Find :

(i) C

(ii) Joint p.d.f. of $U = 2X$ and $V = 2Y$

(iii) Marginal p.d.f. of U and V

(iv) Are U and V independent ? 10

4. (A) Define the chi-square statistic. State its p.d.f. Find mode of a chi-square distribution. State and prove additive property of chi-square distribution.

(B) Define Fisher's t. Derive its p.d.f. 5+5

OR

(E) Define F-statistic. Derive its p.d.f. Find μ'_r and hence find mean and variance of F-distribution. 10

5. Solve any **TEN** of the following questions :

(A) Define Fisher's t.

(B) Show that student's t follows Fisher's t with $(n - 1)$ d.f.

(C) What is mode of t-distribution ?

(D) State conditional mean of X given $Y = y$ if (X, Y) follows bivariate normal distribution.

(E) State m.g.f. of tri-nomial distribution.

(F) State conditional variance of $Y/X = x$ of bi-variate normal distribution.

(G) Define a random sample.

(H) If joint p.d.f. of two r.v.s. of (X, Y) is given by $f(X, Y) = h(X).g(Y)$ where $h(X)$ and $g(Y)$ are non-negative functions of X alone and Y alone respectively, then show that r.v.s X and Y are Stochastically independent.

(I) Define m.g.f. of bivariate probability distribution.

(J) Define $(r, s)^{th}$ product moment about origin. How can it be obtained from the m.g.f. of bivariate probability distribution ?

(K) Let a r.v. $X \sim N(\mu, \sigma^2)$ then derive the distribution of $Z = \left(\frac{X - \mu}{\sigma}\right)$.

(L) Define a statistic. 1×10=10