Bachelor of Arts (B.A.) Fifth Semester Examination

MATHEMATICS

(Metric Space, Complex Integration and Algebra)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that every infinite subset of a countable set is countable. 6

(B) Define a metric on a set X. Determine whether the function d defined by

$$d(x, y) = |x^2 - y^2| \ \forall \ x, y \in R^1$$

is a metric or not. 6

OR

(C) Prove that the set G is open if and only if G^C (Complement of G) is closed. 6

(D) Let E be a non-empty set of real numbers which is bounded above. Let y = sup E. Then prove that $y \in \overline{E}$. 6

UNIT—II

2. (A) Prove that closed subsets of compact sets are compact. Hence or otherwise prove that $A \cap B$ is compact if A is closed and B is compact. 6

(B) Prove that if E is a connected subset of the real line R^1 and $x, y \in E$ such that $x < z < y$, then $z \in E$. 6

OR

(C) Let Y be a subspace of a complete metric space X. Prove that Y is complete if and only if Y is closed. 6

(D) If $\{I_n\}$ is a sequence of intervals in R^1, such that $In \supset I_{n+1}$ (n = 1, 2, 3, ...), then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty. 6

UNIT—III

3. (A) If R is a ring and $a^2 = a$ for all $a \in R$ then prove that R is a commutative ring. 6

(B) Prove that a finite integral domain is a field. 6

OR

(C) Define kernel of a ring homomorphism. If ϕ is a homomorphism of a ring R into a ring R′ with Kernel $I(\phi)$, then prove that $I(\phi)$ if an ideal of R. 6

(D) Let R and R′ be the rings and ϕ be a homomorphism of R onto R′ with Kernel U. Prove that R′ is isomorphic to R/U. 6

UNIT—IV

4. (A) Evaluate $\oint_C \frac{z-1}{z^2(z-2)}\,dz$ using Cauchy's integral formula, where C is the circle $|z - i| = 2$. 6

(B) Verify Cauchy's theorem for $\int_C z^3\,dz$ over the boundary of the triangle with vertices (1, 2), (1, 4), (3, 2). 6

OR

(C) Evaluate $\int_C \frac{z^3}{(z-1)^4(z-2)(z-3)}\,dz$ by the method of calculus of residues, where C is a circle $|z| = 4$. 6

(D) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$. 6

Question—V

5. (A) Prove that the set of even natural numbers is countable. 1½

(B) Define the following :

(i) Derived set

(ii) Closure of a set. 1½

(C) Determine whether the set A = (0, 1) is compact in R. 1½

(D) Define K-cell and explain 1-cell. 1½

(E) If ϕ is a homomorphism of a ring R into a ring R′, then prove that for $a \in R$, the additive inverse of $\phi(a)$ is $\phi(-a)$. 1½

(F) Find the Kernel of a homomorphism $\phi : J(\sqrt{2}) \to J(\sqrt{2})$ defined by $\phi(a + b\sqrt{2}) = a - b\sqrt{2}$ where $J(\sqrt{2})$ is a ring of real numbers of the form $a + b\sqrt{2}$ (a and B are integers). 1½

(G) Examine whether Cauchy's integral theorem is applicable to evaluate $\int_C \frac{z^2+2}{z+2}\,dz$ where C is the circle $|z| = 3$. 1½

(H) Find the residue of $\frac{1}{(z^2+1)^3}$ at z = i. 1½