TKN/KS/16/5966

Bachelor of Computer Application (B.C.A.) Part—I (Semester—II) (C.B.S.) Examination

DISCRETE MATHEMATICS—II

Paper—IV

Time—Three Hours] [Maximum Marks—50

Note :— (1) All questions are compulsory and carry equal marks.

(2) Draw neat, labelled diagrams wherever necessary.

EITHER

1\. (a) Prove that $A - (A - B) \subseteq B$ where A and B are sets. 5

(b) Prove that :

Let R be an equivalence relation on A and let P be the collection of all distinct relative sets R(a) for a in A. Then P is a partition of A and R is an equivalence relation determined by P. 5

OR

(c) Prove that :

$$A - B = A \cap \overline{B}$$

for the set A and B. 5

(d) Let A = {a, b, c, d} and Let R be the relation on A that has the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the diagraph of R and list indegrees and out-degree of all vertices. 5

EITHER

2. (a) Let $f : A \to B$ and $g : B \to C$ be the invertible function then prove that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

5

(b) Prove by Mathematical Induction,

$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

for $r \neq 1$. 5

OR

(c) Let $f : A \to B$ and $g : B \to A$ be functions such that $g \circ f = I_A$ and $f \circ g = I_B$, Then prove that f is

(d) Prove that a tree with n vertices has n – 1 edges. 5

5. (a) Define Symmetric difference of two sets, complement of Set B with respect to A and find $A \oplus B$, $B \oplus C$, if

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{1, 2, 4, 6, 8\}$, $B = \{2, 4, 5, 9\}$,

$C = \{x : x \text{ is positive integer and } x^2 \leq 16\}$. 2½

(b) Let A = {1, 2, 3, 4, 5, 6}

Compute :

$(4, 1, 3, 5) \circ (5, 6, 3)$ 2½

(c) Define :

(i) Distributive Lattice

(ii) Complemented Lattice. 2½

(d) Define Graph. Explain the representation of graph in memory. 2½ one-to-one correspondence from A to B and g is one-to-one from B to A and each is inverse of other. 5

(d) Let A = {1, 2, 3, 4, 5, 6, 7, 8}, determine whether the permutation is even or odd,

(i) (6, 4, 2, 1, 5)

(ii) $(4, 8) \circ (3, 5, 2, 1) \circ (2, 4, 7, 1)$ 5

EITHER

3. (a) Let $f : S \rightarrow T$ be a homomorphism of the semigroup $(S, *)$ onto the semigroup $(T, *)$. Let R be the relation on S defined by aRb if and only if $f(a) = f(b)$ for a and b in S. Then prove that R is a congruence relation. 5

(b) Let A = {1, 2, 3, 4, 12}. Consider a partial order divisibility on A. Draw the Hasse diagram of the poset $(A, \leq)$. 5

OR

(c) Let G be the set of all non zero real numbers and let

$$a * b = \frac{\quad}{2} \quad \forall\ a, b \in G$$

Show that $(G, *)$ is an abelian group. 5

(d) Let L be a bounded distributive lattice. Prove that if complement of $a \in L$ exists, then it is unique. 5

EITHER

4. (a) Define Euler path and Euler circuit. Prove that if a graph G has more than two vertices of odd degree, then there can be no euler path in G. 5

(b) Define :

(i) Tree

(ii) Height of tree

(iii) Complete binary tree. 5

OR

(c) Let number of edges of graph G be m, then prove that G has a Hamiltonian circuit if

$m \geq \frac{1}{2}(n^2 - 3n + 6)$ where n is the number of vertices. 5