Faculty of Engineering & Technology

First Semester B.E. Examination

APPLIED MATHEMATICS—I

Paper—I

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) Solve **SIX** questions as follows :

Que. 01 OR Que. 02, Que. 03 OR Que. 04,

Que. 05 OR Que. 06, Que. 07 OR Que. 08,

Que. 09 OR Que. 10, Que. 11 OR Que. 12.

(2) Use of non-programmable calculator is permitted.

1. (a) If $y = e^{\tan^{-1} x}$, prove that :

$$(1 + x^2) y_{n+2} + [2(n + 1)x - 1] y_{n+1} + n(n + 1) y_n = 0.$$ 6

(b) Evaluate :

(i) $\lim_{x \to 0} \left[\frac{1}{x} - \cot x\right]$ 3

(ii) $\lim_{x \to 0} (1 + \sin x)^{\cot x}$

OR

2. (a) For the curve $(a + x) y^2 = ax$, if ρ is the radius of curvature at a point (x, y), show that

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 \qquad 6$$

(b) Using Taylor's series find the value of cos 64° correct upto four decimal places 6

3. (a) Find the value of n, so that the equation $v = r^n (3\cos^2\theta - 1)$ satisfies the relation :

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial v}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial v}{\partial \theta}\right) = 0. \qquad 6$$

(b) If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. 6

www.rtmnuonline.com

(c) If $u = \log\left[\frac{x^4 - y^4}{x - y}\right]$

prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3. \qquad 6$$

OR

4. (a) Given $u = 4x^2 + 9y^2 + 16z^2$, $v = 12xy - 16xz + 24yz$, $w = 2x + 3y + 4z$. Are u, v, w functionally related ? If so, find the relation between them. 6

(b) Expand $x^2y + 3y - 2$ in power of $(x - 1)$ and $(y + 2)$. 6

(c) A rectangular box, open at the top is to have a volume of 32 C.C. Find the dimensions of the box requiring least material for its construction. 6

5. (a) Find the inverse of matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \text{ by Partitioning} \qquad 7$$

www.rtmnuonline.com
www.rtmnuonline.com

(b) Test the consistency and solve :

$5x + 3y + 7z = 4,$

$3x + 26y + 2z = 9$ and

$7x + 2y + 10z = 5$ 5

OR

6. (a) By adjoint method solve the system of equations

$5x + 3y + 3z = 48,$

$2x + 6y - 3z = 18,$

$8x - 3y + 2z = 21$ 6

(b) Find the Rank of the matrix :

$$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}.$$ 6

7. (a) $\dfrac{dy}{dx} = \dfrac{y+1}{(y+2)e^{y} - x}$ 4

(b) $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$ 4

(c) $\dfrac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ 4

OR

www.rtmnuonline.com

8. (a) Solve

$p^2 + 2py\cot x = y^2$ 3

(b) Solve :

$y = 2px + y^2p^3$ 3

(c) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series is :

$E = Ri + \int \frac{i}{C}\,dt$, find the current i at any time t when $E = E_m \sin \omega t$. 6

9. (a) Solve

$\dfrac{d^4y}{dx^4} - y = \cos x \cosh x$. 6

(b) Solve by method of variation of parameters :

$\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$ 6

(c) $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2\sin(\log x)$ 6

OR

www.rtmnuonline.com

www.rtmnuonline.com

10 (a) Solve the simultaneous differential equations

$$\frac{dx}{dt} + y = \sin t, \quad \frac{dy}{dt} + x = \cos t,$$

given that $x = 2$ and $y = 0$ when $t = 0$. 6

(b) Solve $\frac{d^2y}{dx^2} = \sec^2 y \tan y$, given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$. 5

(c) A spring for which stiffness K = 700 N/m hangs in a vertical position with its upper end fixed. A mass of 7 kg is attached to the lower end. After coming to rest, the mass is pulled down 0.05 m and released. Discuss the resulting motion of the mass, neglecting air resistance. 7

11. (a) Find all the values of $(16)^{1/4}$ in (a + ib) form. 4

(b) If $\cos(\theta + i\phi) = \cos\alpha + i \sin\alpha$, prove that :

$$\cos 2\theta + \cosh 2\phi = 2.$$ 4

OR

12 (a) Express $\sin 7\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$. 4

(b) If $\tan(\theta + i\phi) = \cos\alpha + i \sin\alpha$, prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4}$$ 4