B.E. First Semester (Fire Engineering) (C.B.S.)

# Applied Mathematics – I Paper – I

P. Pages : 4 | **TKN/KS/16/7284**

Time : Three Hours | Max. Marks : 80

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Notes :
1. Solve **six** questions as follows.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

**1.** a) If $y = \sin^{-1} x$ then prove that $\left(1 - x^2\right) y_{n+2} - (2n+1)\, x\, y_{n+1} - n^2 y_n = 0$. **6**

b) Evaluate

1) $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$ **3**

2) $\lim\limits_{x \to 0} \left( \dfrac{a^x + b^x + c^x}{3} \right)^{1/x}$ **3**

**OR**

**2.** a) Prove that for the ellipse **7**

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \quad \rho = \dfrac{a^2 b^2}{P^3}$

Where P is the length of perpendicular from the center upon the tangent at (x, y).

b) Expand log cos x in ascending power of x upto and including the term $x^4$ using taylor's series. **5**

**3.** a) If $x^x y^y z^z = c$ show that at $x = y = z$ **6**

$\dfrac{\partial^2 z}{\partial x\, \partial y} = -(x \log e x)^{-1}$

b) If $u = \sin^{-1}\left[ \dfrac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}} \right]$ then find the value of **6**

$x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x\, \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$

c) If $\phi = f(x, y, z)$ and $x = \sqrt{vw}$ $y = \sqrt{wu}$ $z = \sqrt{uv}$, then show that **6**

$$u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w} = x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z}$$

**OR**

**4.** a) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ **6**

Find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

b) Expand $e^x \sin y$ in the power of x and y upto third degree term. **6**

c) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$ **6**

Find the highest temperature on the surface $x^2 + y^2 + z^2 = 1$.

**5.** a) Find the inverse of matrix by partitioning. **6**

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \end{bmatrix}$$

b) Test the consistency and solve **6**

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

**OR**

**6.** a) Find the rank of matrix **5**

$$\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

b) Solve the system of Equation by Adjoint method. **7**

$$3x + y + z = 8$$
$$2x - 2y + 3z = 7$$
$$x - y + 2z = 5$$

**7.** a) Solve **4**

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$$

b) Solve **4**

$$\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y(\log y)^2}{x^2}$$

c) Solve **4**

$$\left(1+e^{x/y}\right)dx+\left(1-\frac{x}{y}\right)e^{x/y}\,dy=0.$$

**OR**

**8.** a) Solve $xy^2\left(p^2+2\right)=2py^3+x^3$. **3**

b) Solve $y=2px+p^4x^2$. **3**

c) A resistance R = 50 ohms and an inductance L = 10 henries are connected in series with a constant voltage E = 100 volts If the current is zero when t = 0. **6**
Find
a) The equation for i, $E_R$ and $E_L$.
b) The current at t = 0.5 sec.
c) The time at which $E_R$ = L.
Where $E_R$ – voltage across resistance
$E_L$-voltage across inductance.

**9.** a) Solve **6**

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\cos x.$$

b) Solve using method of variation of parameter **6**

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}.$$

c) Solve **6**

$$\frac{d^2y}{dx^2}=3\sqrt{y}\ \text{ given that }\ y=1,\frac{dy}{dx}=2\text{ when } x=0.$$

**OR**

**10.** a) Solve the simultaneous differential equation **6**

$$\frac{d^2x}{dt^2}=b\frac{dy}{dt};\ \frac{d^2y}{dt^2}=a-b\frac{dx}{dt}.$$

b) Solve **6**

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right).$$

c) In an L-C-R circuit the charge q on a plate of a condenser is given by **6**

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=E\sin pt.$$

The circuit is tuned to resonance so that $P^2=\frac{1}{LC}$. If initially current i and charge q be zero. Show that for small value at R/L the current at time t is $\frac{Et}{2L}\sin pt$.

**11.** a) Solve the equation with the help of De Moivre's theorem $x^7 - 1 = 0$. **4**

b) If $2\cos\theta = x + \frac{1}{x}$ **4**

$2\cos\phi = y + \frac{1}{y}$

then prove that

$x^m y^m + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$

**OR**

**12.** a) Find all the values of **4**

$(16)^{1/4}$.

b) If $\cos(\theta + i\phi) = \cos\alpha + i\sin\alpha$ **4**

Then prove that

$\sin^2\theta = \pm\sin\alpha$.

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