B.E. All Branches First Semester (C.B.S.) / B.E. (Fire Engineering) First Semester

Applied Mathematics - I

P. Pages : 3 **NRJ/KW/17/4336**

Time : Three Hours * 0 9 6 8 * Max. Marks : 80

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $y = a\cos(\log x) + b\sin(\log x)$, then show that **6**
$x^2 y_{n+2} + (2n+1)\,x y_{n+1} + (n^2+1)\,y_n = 0$.

b) Evaluate

i) $\lim\limits_{x\to 0} \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x}$ **3**

ii) $\lim\limits_{x\to 0}\left[\dfrac{a^x + b^x + c^x}{3}\right]^{1/x}$ **3**

OR

2. a) Use Taylor's expansion to evaluate sin60º, 30' correct to five decimal places. **6**

b) If $x = a\cos^4\theta,\ y = a\sin^4\theta$, find the radius of curvature at $\theta = \pi/6$. **6**

3. a) If $u = \log(\tan x + \tan y + \tan z)$ then prove that **6**
$\sin 2x \dfrac{\partial u}{\partial x} + \sin 2y \dfrac{\partial u}{\partial y} + \sin 2z \dfrac{\partial u}{\partial z} = 2$

b) **6**
If $u = \log\left(\dfrac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right)$, then show that
$x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x\,\partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = -\dfrac{3}{2}$.

c) If $u = \lambda(x-y,\ y-z,\ z-x)$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$ **6**

OR

4. a) If $u = 3x + 2y - z,\ v = x - 2y + z,\ w = x(x + 2y - z)$, show that they are functionally related and find the relation between them. **6**

b) Expand $e^x \cos y$ in power of x and $\left(y - \pi/2\right)$ upto terms of degree 3. **6**

c) Divide 24 into three parts such that continued product of first, square of second and cube of third is maximum. **6**

5. a) Solve the system of equation by matrix method. **6**

$$x - 2y + 3z = 2$$
$$2x - 3z = 3$$
$$x + y + z = 0$$

b) Find the rank of the matrix **6**

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

OR

6. a) Test the consistency of the following system and solve it **6**

$$x - 2y - z = 5$$
$$x + 8y - 3z = -1$$
$$2x + y - 3z = 7$$

b) By suitable partitioning find the inverse of the matrix A, **6**

where $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 5 & 2 & 3 & -1 \\ -1 & 1 & -5 & 2 \end{bmatrix}$

7. Solve :

a) $\dfrac{dy}{dx} - \dfrac{\tan y}{1+x} = (1+x)e^x \sec y$ **4**

b) $\dfrac{2y}{x}dx + (2\log x - y)\,dy = 0$ **4**

c) $(x+1)\dfrac{dy}{dx} - y = e^x(x+1)^2$ **4**

OR

8. a) An inductance of 2 henries and a resistance of 20 ohms are connected in series with an emf E volts. If the current is zero when t = 0, find the current at the end of 0.01 sec. if E = 100 volts. **4**

b) Solve $p^2 + 2px + py + 2xy = 0$ **4**

c) Solve $y = x + 2\tan^{-1} y$ **4**

9. a) Solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = e^{2x} + \sin 3x$ **6**

b) Solve by method of variation of parameter **6**
$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

c) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 3y = x\log x$ **6**

OR

10. a) Solve $\frac{d^2y}{dx^2} = -\mu\left[y + \frac{a^4}{y^3}\right]$ given $y = 0, \frac{dy}{dx} = 0$ when $x = 0$. **6**

b) Solve the Simultaneous differential equations **6**
$\frac{dy}{dx} - 7x + y = 0;\ \frac{dy}{dx} - 2x - 5y = 0$

c) The differential equation for a circuit in which self inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2} + \frac{i}{c} = 0$, find the current 'i' as a function of t given that I is the maximum current and i = 0 when t = 0. **6**

11. a) Prove that $\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}$. **4**

b) Using De-Moivre's theorem, solve $x^7 + x^4 + x^3 + 1 = 0$ **4**

OR

12. a) If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$ prove that **4**
$2\theta = n\pi + \frac{\pi}{2} + \alpha$.

b) Find the general value of $\log(-i)$. **4**

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