B.E. All Branches Second Semester (C.B.S.) / B.E. (Fire Engineering) Second Semester

Applied Mathematics-II

P. Pages : 3 | NRT/KS/19/3287/3941

Time : Three Hours | Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) Evaluate $\int_0^{\infty} e^{-x^2} \cdot x^2 \cdot dx$. **6**

b) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} dx$, by using differentiation under integral sign, $a > 0$. **6**

OR

2. a) Evaluate $\int_0^1 \frac{x}{\sqrt{1-x^4}} dx$ **6**

b) Show that the RMS value of the function $f(t) = A\sin(pt)$ over half period exceeds its mean value over the same half period by nearly 11%. **6**

3. a) Trace the curve $ay^2 = x^2(a-x)$ **6**

b) Find the perimeter of the cardioid $r = a(1+\cos\theta)$. **6**

OR

4. a) Find the area lying between the parabolas $y^2 = 4x$ and $y^2 = -4(x-2)$. **6**

b) If S is the arc of the curve $y^2 = x\left(1-\frac{x}{3}\right)^2$ measured from the origin to the point (x, y), then show that $S^2 = y^2 + \frac{4}{3}x^2$. **6**

5. a) Evaluate $\int_0^1\int_0^y x\,y\,e^{-x^2}\,dx\,dy$. **6**

b) Evaluate $\int_0^a\int_y^a \frac{x^2}{\left(x^2+y^2\right)^{3/2}}\,dy\,dx$ by changing into polar coordinates. **6**

c) Evaluate by changing the order of integration. **6**

$$\int_0^a\int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}\,dy\,dx.$$

OR

6. a) Evaluate $\iint r^3\,dr\,d\theta$, over the area included between the circles $r=2\cos\theta$ and $r=4\cos\theta$. **6**

b) Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$. **6**

c) Find the centre of gravity of the lamina enclosed by the curves $y=x^2$, $y=0$ and $x=4$ when the density is kx. **6**

7. a) Show that **6**

$$\left(\bar{a}\times\bar{b}\right)\cdot\left\{\left(\bar{b}\times\bar{c}\right)\times\left(\bar{c}\times\bar{a}\right)\right\}=\left(\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)\right)^2.$$

b) Find the directional derivative of $\phi=xy^2+yz^2$ at the point $(2,-1,1)$ in the direction of the vector $i+2j+2k$. **6**

c) A vector field is given by $\bar{F}=(y\sin z-\sin x)i+(x\sin z+2yz)j+\left(xy\cos z+y^2\right)k$. Show that it is irrotational and hence find its scalar potential. **6**

OR

8. a) A particle moves along the curve. **6**

$\bar{r}=\left(t^3-4t\right)i+\left(t^2+4t\right)j+\left(8t^2-3t^3\right)k$, where t is the time. Find the magnitude of tangential and normal components of acceleration.

b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-3=z$ at the point $(2,-1,2)$. **6**

c) If $\overline{A}=\left(x^2-y^2+2xz\right)i+(xz-xy+yz)j+\left(x^2+z^2\right)k$, find curl $\overline{A}$ and show that the vectors given by curl $\overline{A}$ at $(1,2,-3)$ and $(2,3,12)$ are orthogonal. **6**

9. Find the work done in moving a particle once around the circle $x^2+y^2=9, z=0$, under the field of force given by the vector **7**
$\overline{F}=(2x-y+z)i+\left(x+y-z^2\right)j+(3x-2y+4z)k$.

OR

10. Using Gauss divergence theorem, evaluate $\iint_S \overline{F}\cdot\hat{n}\,ds$, where $\overline{F}=4xi-2y^2j+z^2k$. **7**

and S is the surface bounded by the region $x^2+y^2=4, z=0, z=3$.

11. a) Fit a parabola $y=a+bx^2$ using the following data. **7**

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

b) Using Lagrange's interpolation formula, find f (4) from the following data. **6**

x	0	2	3	6
f (x)	-4	2	14	158

OR

12. a) Find the coefficient of correlation and the equations of lines of regression using following data. **7**

x	1	2	3	4	5
y	2	5	3	8	7

b) Solve the difference equation. **6**
$\left(E^2-5E+6\right)y_n=4^n\left(n^2-n+5\right)$.

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