B.E. (Fire Engineering) Third Semester (C.B.S.)

Applied Mathematics – III

P. Pages : 3 **NRT/KS/19/3944**

Time : Three Hours * 0 2 1 8 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$ and hence evaluate $\int_0^\infty \frac{\sin t}{t}\,dt$. **7**

b) Express $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **7**

OR

2. a) Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ by using convolution theorem. **7**

b) Solve by using Laplace transform method. **7**

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t$ subject to the initial condition $y(0) = 0$ & $y'(0) = 0$.

3. a) Prove that $z\{n^p\} = -z\frac{d}{dz}(n^{p-1})$ where P is any positive integer & hence deduce that **6**

$z\{n\} = \frac{z}{(z-1)^2}$ & $z\{n^2\} = \frac{z(z+1)}{(z-1)^3}$.

b) Find Z-transform of $a^n \cos n\theta$ and $a^n \sin n\theta$. **6**

OR

4. a) Use Z-transform to solve the difference equation **6**

$u_{n+2} + 6u_{n+1} + 9u_n = 2^n$

given $u_0 = 0$

$u_1 = 0$

b) Find inverse Z-transform of $\frac{3z^2+2z+1}{z^2-3z+2}$. **6**

5. Find Fourier sine & cosine transform of e^{-ax}. **6**

OR

6. Using the Fourier integral, show that **6**

$$\int_0^\infty \frac{w\sin(xw)}{1+w^2} = \frac{\pi}{2}e^{-x},\ x>0.$$

7. a) If $f(z)$ is analytic function of z, then prove that **6**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

b) If $u = e^x[x\cos y - y\sin y]$. Show that u is harmonic function. Find V such that $f(z) = u+iv$ is an analytic function. **6**

c) Evaluate $\oint \frac{4-3z}{z(z-1)(z-2)}dz$ where C is a circle $|z|=3/2$. **6**

OR

8. a) Expand the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region **6**

i) $|z|<2$

ii) $2<|z|<3$

iii) $|z|>3$ by using Laurent Series.

b) Evaluate $\oint_C \frac{\sin^6 z}{(z-\pi/6)^3}dz$, where C is a circle $|z|=1$. **6**

c) Evaluate $\int_0^\pi \frac{d\theta}{3+2\cos\theta}$ by contour integration. **6**

9. a) Solve in series the equation **8**

$$2x^2\frac{d^2y}{dx^2}+(2x^2-x)\frac{dy}{dx}+y=0$$ by Frobenius method

b) Prove that **4**

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$$

OR

10. a) Solve by Frobenius method **8**

$$4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$$

b) Prove that, **4**

$$J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x \;\&\; J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$$

11. a) Find Fourier Series for $f(x)=x-x^2$ in the interval $-1<x<1$. **6**

b) Solve $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$. **6**

c) Solve $\frac{\partial^2 z}{\partial x^2}-u\frac{\partial^2 z}{\partial y^2}=\frac{4x}{y^2}-\frac{y}{x^2}$. **6**

OR

12. a) Find the Fourier Series for **6**

$f(x)=\begin{cases}\pi+x & -\pi<x\le 0\\ \pi-x & 0\le x<\pi\end{cases}$ & hence

show that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....$

b) Solve the equation $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ given that $u(x,0)=6e^{-3x}$ by the method of separation of variables. **6**

c) Solve $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=24xy$. **6**

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