B.E. (Power Engineering) (New) Third Semester (C.B.S.)

Applied Mathematics – III

P. Pages : 3 **NRT/KS/19/3337**

Time : Three Hours * 0 2 0 7 * Max. Marks : 80

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$, then prove that $L\left\{\int_0^t f(u)\,du\right\} = \frac{\bar{f}(s)}{s}$ & hence find $L\left\{\int_0^t \frac{\sin u}{u}\,du\right\}$. **6**

b) Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$ by using convolution theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$ in terms of unit step function and find its Laplace transform. **6**

b) Solve the eq^n $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ $y(0) = 1$, using Laplace transform. **6**

3. a) Obtain Fourier Series for **6**

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$

Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - - - -$

b) Find the Fourier Sine transform of $f(x) = \frac{\bar{e}^{ax}}{x}$, $a > 0$ **6**

OR

4. a) Obtain half range sine series for $f(x) = \pi x - x^2$ in interval $(0, \pi)$. **6**

b) Express $f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 0 & x \ge \pi \end{cases}$ as Fourier integral and find **6**

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda} \cdot \sin x\lambda\, d\lambda$$

5. a) If $Z\{f(n)\} = F(z)$ then prove that $Z\{n \cdot f(n)\} = -z \cdot \frac{d}{dz} F(z)$ & hence find $Z\left\{n^2\right\}$. **6**

b) Find Z-transform of $\cos n\theta$ & hence find $Z\left\{a^n \cos n\theta\right\}$. **6**

OR

6. a) Find inverse Z-transform of $\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$ using residue method. **6**

b) Solve the difference equation **6**

$y_{n+2} + 5y_{n+1} + 6y_n = 6^n,\ y_0 = 0,\ y_1 = 1$

7. a) If $u = y^3 - 3x^2y$, then show that u is harmonic and find V and corresponding analytic function $f(z) = u + iv$. **6**

b) Evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is circle $|z| = 3/2$, using Cauchy integral formula. **6**

OR

8. a) Find Laurent's series expansion of the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region **6**

i) $1 < |z| < 2$, ii) $|z| > 2$.

b) Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is a circle $|z - i| = 2$ using residue theorem. **6**

9. a) Solve $x(y-z)p + y(z-x)q = z(x-y)$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. **7**

b) Solve using Laplace transform method **7**

$\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = x,\ x > 0, t > 0$

$u(x,0) = 0,\ u(0,t) = 0$

OR

10. a) Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x \partial y^2} - 2\frac{\partial^3 z}{\partial y^2} = \cos(x + 2y)$. **7**

b) Solve using method of separation of variable $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = 3e^{-y} - e^{-5y}$ **7**

when $x = 0$.

11. a) Investigate the linear dependence of vector, if it is linearly dependent, find relation between them **6**

$X_1 = [2\ -1\ 3\ 2],\ X_2 = [1\ 3\ 4\ 2],\ X_3 = [3\ -5\ 2\ 2]$

b) **6**

Find the matrix B which reduce matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to a diagonal form by transformation $B^{-1}AB$. Hence find diagonal form of A.

c) **6**

Give the complete statement of Sylvester's theorem and use it to prove $\log_e^{e^A} = A$, where $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

OR

12. a) **6**

Verify Caley Hamilton theorem for matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 3 & 1 & -1 \end{bmatrix}$ & hence find A^{-1}.

b) **6**

Use matrix method to solve $\frac{d^2x}{dt^2} + 4x = 0$ subject to $x(0) = 1, x'(0) = 0$.

c) **6**

Find largest eigen value and corresponding eigen vector for the matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ by iteration method.

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