

**APPLIED MATHEMATICS - III**

1. Expand  $f(x) = \sqrt{1 - \cos x}$ ,  $0 < x < 2\pi$  in a Fourier Series.

Hence show that 
$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots \infty = \frac{1}{2}. \quad (7)$$

OR

2. If  $f(x) = 2x - x^2$ , expand  $f(x)$  as a Fourier series in the interval  $(0, 3)$  and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots \infty = \frac{\pi}{12}. \quad (7)$$

3. (a) Solve  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$  (5)

(b) Solve 
$$\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y \quad (7)$$

- (c) Using the method of separation of variables, solve

$$\frac{3\partial u}{\partial x} + \frac{2\partial u}{\partial y} = 0, u(x, 0) = 4e^{-x}. \quad (6)$$

OR

4. (a) A tightly stretched string with fixed end point  $x = 0$  and  $x = l$  is initially in a position given by  $y = y_0 \sin^3 \left( \frac{\pi x}{l} \right)$ .

If it is released from rest from this position, find the displacement  $y(x, t)$ . (10)

- (b) Find the vibration  $u(x, y, t)$  of a rectangular membrane  $(0 < x < a, 0 < y < b)$  whose boundary is fixed given that it starts from rest and  $u(x, y, 0) = hxy(a-x)(b-y)$ . (8)

5. Find the curve passing through the points  $(x_1, y_1)$  and  $(x_2, y_2)$  which when rotated about the x-axis, gives a minimum surface area. (7)

OR

6. Find the plane curve of fixed perimeter and maximum area. (7)
7. (a) Are the vectors  $x_1 = (1, 3, 4, 2)$ ,  $x_2 = (3, -5, 2, 2)$  and  $x_3 = (2, -1, 3, 2)$  linearly dependent? If so express one of these as a linear combination of the others. (5)

- (b) Verify Cayley - Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \text{ and find its inverse. Also express } A^5 -$$

$$4A^4 - 7A^3 + 11A^2 - A - 10I \text{ as a linear polynomial in } A. \quad (7)$$

- (c) Use Sylvester's theorem to show that

$$2 \sin A = \{\sin 2\} A, \text{ where } A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

OR

8. (a) Find the modal matrix corresponding to matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix}$$

- (b) Use matrix method to solve the differential equation.

$$\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0, y_{(0)} = 1, y'_{(0)} = 2. \quad (6)$$

- (c) Reduce the quadratic form

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy \text{ to the canonical form.} \quad (6)$$

9. (a) Find by Newton-Raphson method, the real root of  $3x - \cos x - 1 = 0$  correct to four places of decimal. (6)

- (b) Apply Crout's method to solve the equations :

$$3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7. \quad (6)$$

- (c) Using Runge - Kutta method of 4th order, find  $y$  for  $x = 0.1$  &  $0.2$ , given that :

www.rtmnuonline.com  $\frac{dy}{dx} = xy + y^2, y(0) = 1.$

(6)

OR

10. (a) Find a real root of the equation  $x \log_{10} x = 1.2$  by Regula - falsi method correct to four decimal places. (6)
- (b) Apply Gauss-Seidal iteration method to solve the equations :  
 $3x + 20y - z = -18$   
 $2x - 3y + 20z = 25$   
 $20x + y - 2z = 17.$  (6)
- (c) Apply Milne's Predictor - Corrector method, to find a solution of the differential equation  $y' = x - y^2$  in the range  $0 \leq x \leq 1$  for the boundary conditions  $y = 0$  at  $x = 0.$  (6)
11. A company manufactures two types of cloth, using three different colours of wool, one yard length of type A cloth requires 4 m. of red wool, 5 m of green wool and 3 m of yellow wool. One yard length of type B cloth requires 5 m of red wool, 2 m of green wool and 8 m of yellow wool. The wool available for manufacture is 1000 m of red wool, 1000 m of green wool and 1200 m of yellow wool. The manufacturer can make a profit of Rs. 5 on one yard of type A cloth and Rs. 3 on one yard of type B cloth.
- (a) Formulate the problems as a standard L.P.P. (6)
- (b) Find product mix that would give maximum profit by graphical technique. (6)

OR

12. Solve the following L.P.P. by simplex method.

Maximize  $Z = 5x_1 + 3x_2$

Subject to  $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0.$

(12)

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