Third Semester B. E. (Civil Engg.) (C.B.S.) Examination

APPLIED MATHEMATICS – III

Paper–III

Time : Three Hours]   [Max. Marks : 80

N. B. : (1) All questions carry marks as indicated.
(2) Solve **SIX** questions as follows :
Q. No. 1 OR Q. No. 2.
Q. No. 3 OR Q. No. 4.
Q. No. 5 OR Q. No. 6.
Q. No. 7 OR Q. No. 8.
Q. No. 9 OR Q. No. 10.
Q. No. 11 OR Q. No. 12.
(3) Use of non programmable calculator is permitted.

1. (a) Sketch the function

$$f(x) = \begin{cases} \pi + x \ ; & -\pi < x \leq 0 \\ \pi - x \ ; & 0 \leq x < \pi \end{cases}$$

and hence find Fourier series for f(x). Hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$ 7

OR

2. Obtain Fourier series for

$$f(x) = \begin{cases} \pi x \ ; & 0 \leq x \leq 1 \\ \pi\ (2 - x) \ ; & 1 \leq x \leq 2 \end{cases}$$ hence

Show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$ 7

3. (a) Solve $xq = yp + x\, e^{(x^2 + y^2)}$ 5

(b) Solve :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x\sin y$$ 7

(c) Solve using method of separation of variables,

$$3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0 \ , \ u(x, 0) = 4e^{-x}$$ 6

OR

4. (a) A Tightly stretched string with fixed end points $x = 0$, $x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(l - x)$, find the displacement of the string at any distance from one end at any time t. 8

(b) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y} + \sin(x - 2y)$. 5

(c) Solve $y^2 p - xyq = x(z - 2y)$. 5

5. Find the extremal of the functional

$$\int_{x_0}^{x_1} \{x^2 (y')^2 + 2y^2 + 2xy\} \, dx$$ 7

OR

6. Find the plane closed curve of fixed perimeter and maximum area. 7

7. (a) Show by matrix, the vectors X_1 [2, 3, 1, –1], X_2 [2, 3, 1, –2], X_3 [4, 6, 2, –3] are linearly dependent. Find the relation between them. 5

(b) Reduce the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix}$$

to the diagonal form. 6

(c) Verity Cayley – Hamilton theorem for given matrix A and hence find A^{-2}

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$ 7

OR

8. (a) Use Sylvester's theorem to show that 3tan A = (tan 3)A, where

$$A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$$ 6

(b) Solve the following differential equation by using matrix method

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0, \text{ given } y(0) = 2,$$

$y'(0) = 5$ 6

(c) If $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 7 & 4 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Express x_1, x_2, x_3 in terms of z_1 and z_2. 6

9. (a) Find the root of equation $x \log_{10} x - 1.2 = 0$ by method of false position, correct upto four places of decimal. 6

(b) Apply Gauss – Seidal iteration method to solve the equations

$$2x - 3y + 20z = 25,$$
$$20x + y - 2z = 17,$$
$$3x + 20y - z = 18.$$ 6

(c) Use Runge–Kutta method to find approximate value of y for x = 0.2, when

$$\frac{dy}{dx} = x - 2y, \text{ given } y(0) = 1, \ h = 0.1$$ 6

OR

10. (a) Write Newton– Raphson formula for finding $\sqrt[3]{N}$, where N is a real number. Use it to find $\sqrt[3]{18}$ by assuming 2.5 as initial appr-oximation. 6

(b) Use Crout's method to solve the equations
$5x + 2y + z = 12,$
$x + 4y + 2z = 15,$
$x + 2y + 5z = 20.$ 6

(c) By Milne's predictor–corrector method

$$\frac{dy}{dx} = \frac{1}{x+y} \ ; \ y(0) = 2, \ y(0.2) = 2.0933,$$

$y(0.4) = 2.1755$, $y(0.6) = 2.2493$, find $y(0.8)$. 6

11. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs 5/kg and B_2 costs Rs 8/kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find graphically the minimum cost of the brick satisfying the above conditions. 12

OR

12. Solve the following L.P.P.
Maximize $Z = 12x_1 + 15x_2 + 14x_3$
subject to
$-x_1 + x_2 \leqslant 0$
$-x_2 + 2x_3 \leqslant 0$
$x_1 + x_2 + x_3 \leqslant 100$
$x_1, x_2, x_3 \geqslant 0$ 12