B.E. (Civil Engineering) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NRT/KS/19/3292**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of graph paper is permitted.

1. Obtain the Fourier series for **7**

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi}, & -\pi<x<0\\ 1-\dfrac{2x}{\pi}, & 0<x<\pi\end{cases}$$

Hence deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$.

OR

2. Obtain half range sine series for $f(x)=\pi x-x^2$ in the interval $(0, \pi)$. **7**

3. a) Solve: $x\left(y^2-z^2\right)p+y\left(z^2-x^2\right)q=z\left(x^2-y^2\right)$. **6**

b) Solve: $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial x\partial y}=\cos x-\cos 2y$. **6**

c) Solve the following by method of separation of variables: **6**
$\dfrac{\partial u}{\partial x}=4\dfrac{\partial u}{\partial y}$. Given $u(0,y)=8e^{-3y}$.

OR

4. a) A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in a position given by $y=y_0\sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement y (x, t). **7**

b) Solve by using method of separation of variables $3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0$, given that $u(x, 0) = 4e^{-x}$. **5**

c) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. **6**

5. Find the extremals of the functional **7**

$$V[y(x)] = \int_{x_0}^{x_1} \left[16y^2 - y''^2 + 2ye^x\right] dx.$$

OR

6. Find the curve passing through the points (x_1, y_1) and (x_2, y_2), which when rotated about x-axis gives minimum surface area. **7**

7. a) Investigate the linear dependence of the vectors $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$ & $x_4 = (-3, 7, 2)$. and if possible, find relation between them. **6**

b) Reduce the matrix $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ to the diagonal form. **6**

c) Using Sylvester's theorem, verify. **6**

$\log_e e^A = A$, if $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

OR

8. a) Verify Cayley-Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **6**

b) Solve by matrix method $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ given $y(0) = 2$, $y'(0) = 5$. **6**

c) Diagonalize the matrix by orthogonal transformation. **6**

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

9. a) Using Newton Raphson method, find the root of the equation $x^4 - x - 9 = 0$, correct upto 3 places of decimal. **6**

b) Solve the system of equation by Gauss-Seidal Method. **6**
$4x + 11y - z = 33,\ 6x - 3y + 12z = 36;\ 8x - 3y + 2z = 20.$

c) Given $\frac{dy}{dx} = -xy^2,\ y(0) = 2,\ h = 0.1$, find y when x = 0.2, using Euler's modified method. **6**

OR

10. a) Find by False position method the root of the equation $xe^x = \cos x$. **6**

b) Solve by 4th order Runge-Kutta method $\frac{dy}{dx} = 3x + y^2$, given $y = 1.2$, when x=1, find $y(1\cdot1)$. **6**

c) Solve system of equation by Crout's method **6**
$4x + 11y - z = 33;\ 6x + 3y + 12z = 36;\ 8x - 3y + 2z = 20.$

11. Use Simplex method to solve the following L.P.P: **12**

Maximize $z = 5x + 3y$

s.t. $2x + 4y \le 12$

$2x + 2y = 1$

$5x + 2y \ge 10$

$x, y \ge 0$

OR

12. a) A farmer wants to make sure that his herd get the minimum daily requirement of three basic nutrient A, B, C. Daily requirement are 15 units of A, 20 units of B and 30 units of C one gram of product P has 2 units of A, 1 unit of B and 1 unit of C. One gram of product Q has 1 unit of A, 1 unit of B and 3 units of C. The cost of P is Rs. 12/gram and cost of B is. Rs. 18/gram. Formulate the l.p.p. to minimize cost. **6**

b) Solve the LPP using graphical method **6**

Minimize Z = 20 x + 10 y

s.t. $x + 2y \le 40$; $4x + 3y \ge 60$;

$3x + y \ge 3$; $x,\ y \ge 0.$

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