B.E. (Civil Engineering) Third Semester (C.B.S.)

# Applied Mathematics - III

P. Pages : 3 **NRJ/KW/17/4347**

Time : Three Hours * 0 4 9 9 * Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of graph sheet is permitted.

**1.** Find Fourier series for $f(x) = x - x^2, \quad -\pi < x < \pi$. **7**

**OR**

**2.** Find half range cosine series for $f(x) = (x-1)^2, \ 0 < x < 1$. **7**

**3.** a) Solve $z(x+y)p + z(x-y)q = 2x^2 + y$. **6**

b) Solve $\left(D^2 + 2DD' - 8D'^2\right)z = e^{2x+y} + \sqrt{2x+3y}$. **6**

c) Solve $3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0$ given that $u(x,0) = 4e^{-x}$ by method of separation of variables. **6**

**OR**

**4.** a) A tightly stretched string with fixed end points x=0 and x=l, is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(\ell - x)$. Find the displacement of the string at any distance from one end at any time t, if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **8**

b) Solve $z(p-q) = z^2 + (x+y)^2$. **5**

c) Solve $\left(D^2 - 3DD' + 2D'^2\right)z = e^{2x+3y} + \sin(x-2y)$. **5**

**5.** Find the extremal of the functional $\int_a^b \frac{\sqrt{1+(dy/dx)^2}}{x^2}\,dx$ given y(1)=0, y(2)=1. **7**

**OR**

**6.** Find the plane closed curve of fixed perimeter and maximum area. **7**

**7.** a) If $A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$, find the eigen values of $A^3-6A^2+3A-2I$. Also find the spectral radius of the matrix represented by $A^3-6A^2+3A-2I$. **6**

b) Diagonalize the matrix $A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **6**

c) Using Sylvester's theorem show that $e^A=e^x\begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix}$ where $A=\begin{bmatrix} x & x \\ x & x \end{bmatrix}$. **6**

**OR**

**8.** a) Verify Cayley Hamilton theorem and hence find $A^{-1}$, where $A=\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$. **6**

b) Solve by matrix method the equation $\frac{d^2y}{dt^2}-5\frac{dy}{dt}+6y=0$ given y(0)=2, y'(0)=5. **6**

c) Reduce the quadratic form $5x^2+6y^2+7z^2-4xy+4yz$ to canonical form by orthogonal transformation. **6**

**9.** a) Find the root of the equation $3x-\cos x-1=0$ correct to third decimal place by regula falsi method. **6**

b) Solve by using Crout's method. **6**

$$4x+y-z=13$$
$$3x+5y+2z=21$$
$$2x+y+6z=14$$

c) Solve by Runge – Kutta fourth order method $\frac{dy}{dx}=\frac{y-x}{y+x}$, y(0)=1 by taking h=0.2. **6**

**OR**

**10.** a) Using Euler's modified method, solve the equation $\frac{dy}{dx}+xy^2=0$, $y(0)=2$, find y(0.2) taking h=0.1. **6**

b) Find the root of $x\log_{10}x-2=0$ by Newton – Raphson method correct upto three decimal places. **6**

c) Solve by Gauss – Seidel method the following system of equations. **6**

$$x + 7y - 3z = -22$$
$$x - 2y + 3z = 18$$
$$2x - y + 6z = 22$$

**11.** a) A company produces three products $P_1$, $P_2$ and $P_3$ from two raw materials A and B, and labour L. One unit of product $P_1$ requires one unit of A, 3 units of B and 2 units of L. One unit of product $P_2$ requires 2 units of A and B each, and 3 units of L, while one unit of $P_3$ needs 2 units of A, 6 units of B and 4 unit of L. The company has a daily availability of 8 units of A, 12 units of B and 12 units of L. It is further known that the unit contribution margin for the products is Rs. 3, 2 and 5 respectively for $P_1$, $P_2$ and $P_3$. Formulate this problem as a linear programming problem. **6**

b) Solve graphically **6**

maximize $z = 6x_1 + 14x_2$

subject to $5x_1 + 4x_2 \geq 60$

$3x_1 + 7x_2 \leq 84$

$x_1 + 2x_2 \geq 18$ & $x_1, x_2 \geq 0$

**OR**

**12.** Use simplex method to solve the following l.p.p. **12**

maximize $z = 5x + 3y$

sub. to $x + y \leq 2$

$5x + 2y = 10$

$3x + 8y \geq 12$

$x, y \geq 0$

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