B.Tech.(Chemical Engineering) Third Semester (C.B.S.)

Engineering Mathematics - III

P. Pages : 3 | NRT/KS/19/3776

Time : Three Hours | Max. Marks : 80

* 0 0 3 9 *

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) Find Laplace Transform of **6**

$$\int_0^t \frac{2(1-\cos u)}{u}\,du$$

b) A mechanical system with two degrees of freedom satisfies the equations **7**

$$2\frac{d^2x}{dt^2}+3\frac{dy}{dt}=4,\ 2\frac{d^2y}{dt^2}-3\frac{dx}{dt}=0.$$

Using Laplace transform find x and y at any instant, given that $x, y, \frac{dx}{dt}, \frac{dy}{dt}$ are zero when $t = 0$.

OR

2. a) Using convolution theorem find $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right]$ **7**

b) Express **6**

$$f(t)=\begin{cases}\sin t, & 0<t<\pi\\ \sin 2t, & \pi<t<2\pi\\ \sin 3t, & t>2\pi\end{cases}$$

in terms of unit step function and hence find its Laplace transform.

3. a) Find Z – transform of sin (3n+5) and cos (3n+5) **6**

b) Find inverse Z-transform of $\frac{z^3}{(z-2)^3}, |z|>2$ **7**

by using power series method

OR

4. a) Find inverse Z-transform of $\frac{z-4}{(z-1)\,(z-2)^2}$, by using partial Fraction method. **6**

b) Solve using Z-transform $y_{n+2} - 2\cos\alpha \cdot y_{n+1} + y_n = 0$ given $y_0 = 0$ and $y_1 = 1$. **7**

5. a) Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$ **7**

b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. **6**

OR

6. a) Solve $z^2(p^2x^2 + q^2) = 1$ **6**

b) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$ **7**

7. a) Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$ by method of separation of variables. **6**

b) The ends A and B of a rod of length ℓ are kept at 0°C and 100°C respectively until the steady state conditions prevail. If the temperature of end A is suddenly raised to 20°C and that of B is reduced to 80°C and kept so. Find the temperature distribution at a distance x from A at any time t. **8**

OR

8. a) A tightly streached string with fixed end points x =0, x = ℓ, is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(\ell - x)$, find the displacement of the string at any distance from one end at any time t. **7**

b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions $u(o, y) = u(\ell, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{\ell}\right)$ **7**

9. a) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with conditions $u(0, t) = u(1, t) = 0$, $u(x, 0) = \frac{1}{2}x(1 - x)$ and $u_t(x, 0) = 0$, taking h = k = 0.1 for $0 \le t \le 0.4$. **7**

b) Solve the equation **7**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions $u(x, 0) = \sin \pi x$, $0 \le x \le 1$; $u(0, t) = u(1, t) = 0$. Carry out computations for two levels, taking $h = 1/3$, $k = 1/36$.

OR

10. a) Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$ for the square mesh with $u(x, y) = 0$ on the four boundaries dividing the square into 16-sub squares of length one unit. **7**

b) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of figure below with boundary values as shown **7**

0	500	1000	500	0
1000	u_1	u_2	u_3	1000
2000	u_4	u_5	u_6	2000
1000	u_7	u_8	u_9	1000
0	500	1000	500	0

11. a) If $u = y^3 - 3x^2y$, then show that u is harmonic and find v and the corresponding analytic function $f(z) = u + iv$. **7**

b) Evaluate **6**

$$\oint_C \frac{\sin^6 z}{\left(z - \pi/6\right)^3} dz$$

where C is the circle $|z| = 1$, by using Cauchy integral formula.

OR

12. a) Evaluate $\oint_C \frac{1}{\sin hz} dz$, where C is a circle $|z| = 4$. **6**

b) Evaluate $\int_0^\infty \frac{dx}{(1 + x^2)^2}$ by contour integration. **7**

****