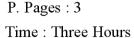
B. Tech. (Chemical Engineering) Third Semester (C.B.S.)

Engineering Mathematics - III





NRT/KS/19/3776

Max. Marks: 80

- Notes:
- 1. All questions carry marks as indicated.
- 2. Solve Question 1 OR Questions No. 2.
- 3. Solve Question 3 OR Questions No. 4.
- 4. Solve Question 5 OR Questions No. 6.
- 5. Solve Question 7 OR Questions No. 8.
- 6. Solve Question 9 OR Questions No. 10.
- 7. Solve Question 11 OR Questions No. 12.
- 8. Use of non programmable calculator is permitted.
- 1. Find Laplace Transform of a)

Laplace Transform of
$$\int_{0}^{t} \frac{2(1-\cos u)}{u} du$$

A mechanical system with two degrees of freedom satisfies the equations b)

$$2\frac{d^2x}{dt^2} + 3\frac{dy}{dt} = 4$$
, $2\frac{d^2y}{dt^2} - 3\frac{dx}{dt} = 0$.

Using Laplace transform find x and y at any instant, given that x, y, $\frac{dx}{dt}$, $\frac{dy}{dt}$ are zero when t = 0.

OR

2. a)

Using convolution theorem find $L^{-1} \left[\frac{1}{s^2(s+1)^2} \right]$

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b) **Express**

$$f(t) = \begin{cases} \sin t \ , & 0 < t < \pi \\ \sin 2t \ , & \pi < t < 2\pi \\ \sin 3t \ , & t > 2 \, \pi \end{cases}$$

in terms of unit step function and hence find its Laplace transform.

- Find Z transform of $\sin (3n+5)$ and $\cos (3n+5)$ 3. a)
 - 7 b) Find inverse Z-transform of $\frac{z^3}{(z-2)^3}$, |z| > 2

by using power series method

OR

- 4. a)
- Find inverse Z-transform of $\frac{z-4}{(z-1)(z-2)^2}$, by using partial Fraction method.

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- b) Solve using Z-transform $y_{n+2} 2\cos\alpha \cdot y_{n+1} + y_n = 0$ given $y_0 = 0$ and $y_1 = 1$.
- 5. a) Solve $(D^3 + D^2D' DD'^2 D'^3)z = e^x \cos 2y$
 - b) Solve $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$.

OR

- 6. a) Solve $z^2(p^2x^2+q^2)=1$
 - b) Solve $(D^2 + 2DD' + D'^2 2D 2D')z = \sin(x + 2y)$
- Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$ by method of separation of variables.
 - b) The ends A and B of a rod of length ℓ are kept at 0°C and 100°C respectively until the steady state conditions prevail. If the temperature of end A is suddenly raised to 20°C and that of B is reduced to 80°C and kept so. Find the temperature distribution at a distance x from A at any time t.

OR

- 8. a) A tightly streached string with fixed end points x = 0, $x = \ell$, is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(\ell x)$, find the displacement of the string at any distance from one end at any time t.
 - Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions $u(o,y) = u(\ell,y) = u(x,0) = 0$ and $u(x,a) = \sin\left(\frac{n\pi x}{\ell}\right)$
- Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with conditions u(0,t) = u(1,t) = 0, $u(x,0) = \frac{1}{2}x(1-x)$ and $u_t(x,0) = 0$, taking h = k = 0.1 for $0 \le t \le 0.4$.
 - b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x, 0) = \sin \pi x, 0 < x < 1; u(0, t) = u(1, t) = 0. Corrected.$

subject to the conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$; u(0,t) = u(1,t) = 0. Carry out computations for two levels, taking $h = \frac{1}{3}$, $k = \frac{1}{36}$.

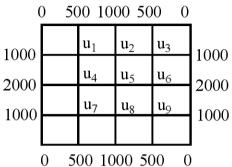
OR

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- Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$ for the square mesh with u(x,y) = 0 on the four boundaries dividing the square into 16-sub squares of length one unit.
 - b) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of figure below with boundary values as shown



11. a) If $u = y^3 - 3x^2y$, then show that u is harmonic and find v and the corresponding analytic function f(z) = u + iv.

where C is the circle |z| = 1, by using Cauchy integral formula.

b) Evaluate $\oint_{C} \frac{\sin^{6} z}{(z - \frac{\pi}{6})^{3}} dz$

OR

- Evaluate $\oint_C \frac{1}{\sin hz} dz$, where C is a circle |z| = 4.
 - Evaluate $\int_{0}^{\infty} \frac{dx}{(1+x^2)^2}$ by contour integration.

