B.E. Third Semester (Computer Science & Engineering (New) / Computer Technology) (C.B.S.)

Applied Mathematics

P. Pages : 3

Time : Three Hours

NKT/KS/17/7232/7237

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Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \overline{f}(s)$, then prove that $L\{f'(t)\} = s\overline{f}(s) - f(0)$ and hence find **6**

$L\left\{\frac{d}{dt}\left(\frac{\sin t}{t}\right)\right\}$.

b) Use convolution theorem to find **6**

$$L^{-1}\left\{\frac{S}{(S+2)(S^2+9)}\right\}$$

OR

2. a) Express $f(t) = \begin{cases} (t-1), & 1 < t < 2 \\ (3-t), & 2 < t < 3 \end{cases}$ **6**

in terms of unit step function and hence find its Laplace transform.

b) Solve **6**

$$f(t) = t^2 + \int_0^t f(u)\sin(t-u)\,du$$

3. a) Find Fourier Series for **6**

$$f(x) = \begin{cases} \pi + x, & -\pi < x \le 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$$

and hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$

b) Solve the integral equation **6**

$$\int_0^\infty f(x)\cos\lambda x\,dx = e^{-\lambda},\ \lambda > 0$$

OR

4. a) Obtain half range cosine series for $f(x) = (2x-1); \quad 0 < x < 1$. **6**

b) Using Fourier integral, show that **6**

$$\int_0^\infty \frac{w \sin(xw)}{1+w^2} dw = \frac{\pi}{2} e^{-x}, \; x > 0$$

5. a) If $Z\{f(n)\} = F(z)$, prove that $Z\{f(n+k)\} = z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i} \right] k > 0$ **6**

and hence find $Z\left\{\frac{1}{(n+1)!}\right\}$.

b) By using convolution theorem find $Z^{-1}\left\{\frac{z^2}{(z-1)(z-3)}\right\}$ **6**

OR

6. a) Find inverse Z-transform of **6**

$$\left\{\frac{z^2 + z}{(z-1)(z^2+1)}\right\}$$

b) Solve $x_{n+2} - 3x_{n+1} + 2x_n = 4^n, \; x_0 = 0, \; x_1 = 1$ **6**

7. a) If $u = y^3 - 3x^2 y$, show that u is harmonic function. Find V and the corresponding analytic function $f(z) = u + iv$. **7**

b) Evaluate using Cauchy's integral formula **7**

$\oint_C \frac{(4-3z)}{z(z-1)(z-2)} dz$, where c is a circle $|z| = 3/2$.

OR

8. a) Find Laurent's series expansion of $f(z) = \frac{z^2 - 4}{(z+1)(z+4)}$ valid for **7**

(i) $|z| < 1$ (ii) $1 < |z| < 4$ and

(iii) $|z| > 4$.

b) Use residue theorem to evaluate $\oint_C \frac{e^{zt}}{z(z^2+1)} dz, \; t > 2$, where C is an ellipse **7**

$\left|z - \sqrt{5}\right| + \left|z + \sqrt{5}\right| = 6$.

9. a) Find whether the following set of vectors are linearly dependent. If so, find relationship. **6**

$X_1 = (1, 2, -1, 3), X_2 = (2, -1, 3, 2)$ and $X_3 = (-1, 8, -9, 5)$.

b) Reduce the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ to the diagonal form. **6**

c) Find the largest eigen value and corresponding eigen vector for the matrix **6**

$A = \begin{bmatrix} 1 & 6 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by iteration method.

OR

10. a) Verify Cayley - Hamilton theorem for the matrix A and find A^{-1}, where **6**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

b) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find A^n, using Sylvester's theorem. **6**

c) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$, given $y(0) = 3,\ y'(0) = 15$ by matrix method. **6**

11. a) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The respective probabilities of an accident are 0.01, 0.03 and 0.15. Out of the insured persons meets an accident. What is the probability that he is a scooter driver? **6**

b) Let X be a random variable having density function **6**

$$f(x) = \begin{cases} cx; & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) the constant C, (ii) $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$ and (iii) the distribution function.

OR

12. a) Find moment generating function and first four moments about the origin for random variable X given by **6**

$$X = \begin{cases} 1/2, & \text{prob.}1/2 \\ -1/2, & \text{prob.}1/2 \end{cases}$$

b) A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts produced by this machine (i) between 30 and 50 and (ii) at the most 30 bolts will be defective. (use normal approximation). **6**

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