B.E. (Computer Science & Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory Paper - I

P. Pages : 4

Time : Three Hours

TKN/KS/16/7376/7381/7386/7391

Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Due credit will be given to neatness and adequate dimensions.
9. Assume suitable data whenever necessary.
10. Illustrate your answers whenever necessary with the help of neat sketches.
11. Use of non programmable calculator is permitted.

1. a) If $U=\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $A=\{1, 5, 6, 7, 8\}$, $B=\{0, 1, 6, 7\}$ $C=\{1, 2, 3, 5, 8\}$, verify that **5**

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $(A \cap B)' = A' \cup B'$

b) Using mathematical induction, show that for all positive integers n **5**
$n^3 - 4n + 6$ is divisible by 3.

OR

2. a) Check the validity of the following argument by truth table: **5**
"If Roli has completed MBA, then she is assured of a good job. If Roli is assured of a good job, she is happy Roli is not happy, so Roli has not completed M. B. A.

b) In a group of students, 70 have a personal computer, 120 have a personal stereo and 41 have both. How many own at least one of these devices? Draw an appropriate Venn diagram also. **5**

3. a) Let A be a set of non-zero integers and let R be a relation on A x A defined by **7**
$(a, b)\, R\, (c, d) \Leftrightarrow ad = bc$, show that R is an equivalence relation.

b) Let $A=\{0, 1, 2, 3\}$, **6**

a) Write the matrices of the relation $R=\{(x, y): x + y = 3\}$ and
$S=\{(x, y): x + y \leq 4\}$

b) Find $M_R \odot M_S$

c) $(M_R \odot M_S)' = M'_S \odot M'_R$

c) Consider A = B = C = R and let $f: A \to B$, $g: B \to C$ be defined by $f(x) = x + 9$ and $g(y) = y^2 + 3$ find $(f \circ g)(3)$, $(g \circ f)(3)$, $(f \circ f)(3)$, $(g \circ g)(3)$. 5

OR

4. a) Let A = {a , b , c} and P(A) be its power set. Let $\subseteq$ be the partial order relation on it. Draw Hasse diagram of $(P(A), \subseteq)$ 6

b) Consider a relation R defined on A as $R = \{(1,2),(2,3),(3,4),(2,1)\}$ and $A = \{1, 2, 3, 4\}$. Find transitive closure of R. Also draw diagraph of transitive A closure relation. 6

c) List all possible functions from set $x = \{a, b, c\}$ to the set $y = \{0, 1\}$ indicate in each case whether the function is one-one, onto or both. 6

5. a) Prove that the set of matrices of the form $\begin{bmatrix} a & o \\ o & a^{-1} \end{bmatrix}$, $a \neq o$, $a \in R$ is a group with respect to matrix multiplication. Is the group abelian? 6

b) Consider a mapping $f: G \to G$ defined by $f(x) = x^4$, where G is a multiplicative group of non-zero complex numbers. Show that the mapping is homomorphism with kernel $f = \{1, -1, i, -i\}$ 6

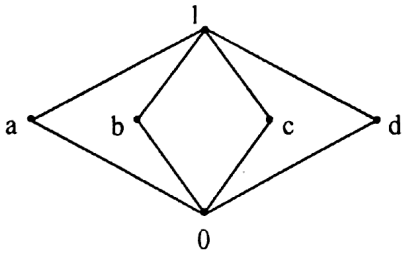
OR

6. a) Prove that cube roots of unity forms a group under multiplication. 6

b) A homomorphism f from G into G′ with Kernel K is an isomorphism of G into G′ *iff* $K = \{e\}$. 6

7. a) Show that the set of integers with the composition 'o' and '*' defined by $a \circ b = a + b + 1$ and $a * b = ab + a + b$ is a ring. 6

b) Define complemented, modular and distributive lattice. Show that the following lattice is complemented, modular but not distributive. 6



OR

8. a) Construct the switching circuit for the boolian expression, and simplify. Construct equivalent simplified circuit also $(A \cdot B) + [A' \cdot (A + B + B')]$. 6

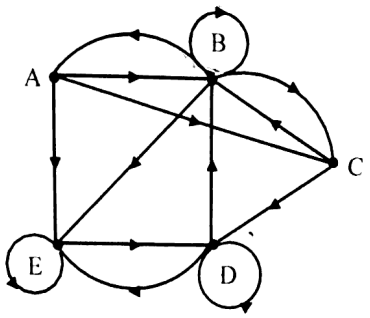
b) If R is ring such that $a^2 = a$, $\forall a \in R$, then show that 6

i) $a + a = 0$, $\forall a \in R$

ii) $a + b = 0 \Rightarrow a = b$, $\forall a, b \in R$

iii) R is commutative ring.

9. a) Define in-degree and out degree of vertex. Find in-degree and out degree of each vertex of the following directed graph. 6



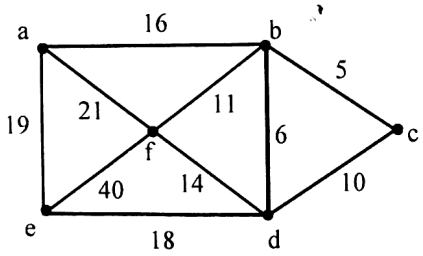
b) Draw a digraph corresponding to the following adjacency matrix and interprete the results 6
AA^T, A^TA, A^2, A^3, A^4.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

c) Draw the binary tree of the expression – 6
$((5*a) + (3-(6*a))) + (a-(3*b))$

OR

10. a) Use prims algorithm to find a minimal spanning tree for the given graph. 6



b) Define : 6

i) Null graph ii) Trial iii) Reachable Node

iv) Tree v) Height of the tree vi) Radius of a graph.

c) Draw the digraph corresponding to matrix. 6

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

find its complement and write the matrix of that complemented graph.

11. a) How many people must you have guarantee that at least g of them will have birthday in the same day of the week. **5**

b) Solve the recurrence relation using generating function $a_n = 3a_{n-1} + 2$, $a_0 = 1$. **5**

OR

12 a) Prove that $c(n+1, r) = c(n, r) + c(n, r-1)$ **5**

b) Use generating function technique to solve following recurrence relation: **5**
$a_n - 9a_{n-1} + 20a_{n-2} = 0$ given $a_0 = 3$, $a_1 = 10$.

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