B.E. (Computer Science Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory

P. Pages : 3 **NRJ/KW/17/4428/4433/4438/4443**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Illustrate your answers whenever necessary with the help of neat sketches.
9. Use of non programmable calculator is permitted.

1. a) Prove by mathematical induction method that $3^{4n+2}+5^{2n+1}$ is a multiple of 14 for $n \geq 0$. **5**

b) Prove the logical equivalence by using algebra of proposition. **5**
$[p \wedge (\sim p \vee q)] \vee [q \wedge \sim (p \wedge q)] = q$

OR

2. a) Check the validity of the argument. **5**
"If the labour market is perfect, then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are note equal. Therefore, the labour market is not perfect.

b) Prove that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology. **5**

3. a) Let X = {ball, bed, dog, let, egg} **6**
& $R = \{(x,y) \mid x, y \in x, xRy$ if x and y contain common letter$\}$
Prove that R is compatible. Is R an equivalence relation?

b) Let $A = Z^{+}$, the set of positive integers and let $R = \{(a,b) \in A x A \mid a \text{ divides } b\}$. **6**
Is R symmetric, asymmetric or antisymmetric?

c) If $f : x \rightarrow y$ and $g : y \rightarrow z$ and both f and g are one-one and onto then show that gof is also one-one onto and $(g \, o f)^{-1} = f^{-1} o g^{-1}$. **6**

OR

4. a) By using the properties of characteristic function prove that **6**
$f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2 f_A(x) \cdot f_B(x)$ for all x.

b) Let $A = \{a, b, c\}$ and $\rho(A)$ be its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hasse diagram of $(\rho(A), \subseteq)$. **6**

c) Let $A=\{1,2,3,4\}$ and $R=\{(1,2),(2,3),(3,4),(2,1)\}$. **6**
Find the matrix of relation and transitive closure of R.

5. a) Show that the fourth root of unity form an abelian group with respect to multiplication. **6**

b) Prove that order of each subgroup of a finite group is a divisor of the order of the group. **6**

OR

6. a) Show that the set of matrices. **6**

$$A\alpha=\begin{bmatrix}\text{Cos}\,\alpha & -\sin\alpha\\ \text{Sin}\,\alpha & \text{Cos}\,\alpha\end{bmatrix},\alpha\in R.$$

forms a monoid.

b) Prove that any two right or left cosets of a subgroup are either disjoint or identical. **6**

7. a) Prove that the set $R=\{0,2,4,6,8\}$ is a commutative ring under addition and multiplication modulo 10. **7**

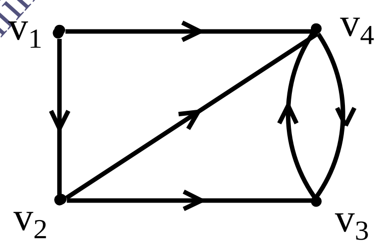
b) Define lattice. Draw the Hasse diagram of lattices D_{24} and D_{30}. **5**

OR

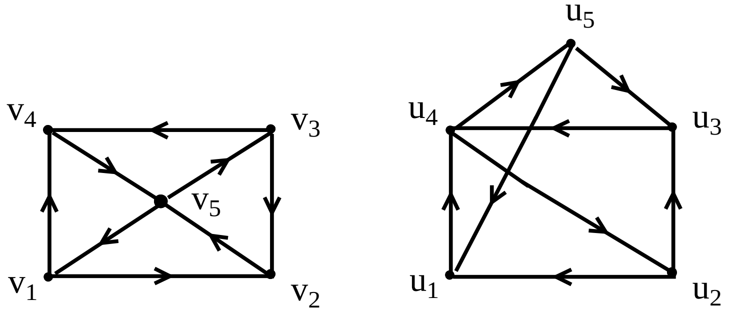
8. a) Construct switching circuit for the Boolean expression $(A\cdot B)+(A\cdot B')+(A'\cdot B')$. **6**
Simplify this and construct an equivalent simplified circuit.

b) Prove that every field is an integral domain. **6**

9. a) Give three different elementary paths from v_1 to v_3 for the digraph given below. What is the shortest distance between v_1 and v_3? Is there any cycle in the graph? **6**



b) Show that the following graphs are isomorphic. **6**



c) Draw the digraph corresponding to matrix. **6**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Find $AA^T, A^TA,$ and A^2 and interpret the result.

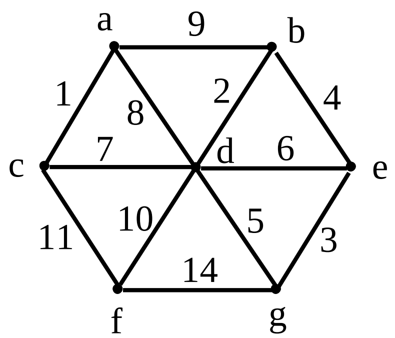
OR

10. a) Draw tree representation for the tree given by R ={(1,2), (1,3), (1,4), (2,5), (4,6), (4,7)} on set A = {1, 2, 3, 4, 5, 6, 7} and draw corresponding binary tree. **6**

b) Define **6**

i) Complete graph.	ii) Euler's path.
iii) Bridge for connected graph	iv) Forest
v) Root of the tree	vi) Regular graph

c) Apply Kruskal's algorithm to construct a minimal spanning tree for the weighted graph given below. **6**



11. a) Prove that. **5**

$c(n+1, r) = c(n, r) + c(n, r-1)$.

b) Find generating function of $n^2, n \geq 0$. **5**

OR

12. a) Find the general solution of the recurrence relation: **5**

$a_{n+2} - 2a_{n+1} + a_n = 2^n,\ a_0 = 2,\ a_1 = 1.$

b) Show that if seven numbers from 1 to 12 are chosen then two of them will add upto 13. **5**

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