Faculty of Engineering & Technology

Third Semester B.E. (Information Technology)/ C.E.(C.B.S.) Examination

APPLIED MATHEMATICS—III

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) Solve **SIX** questions as follows :

Question No. **1** **OR** Question No. **2**

Question No. **3** **OR** Question No. **4**

Question No. **5** **OR** Question No. **6**

Question No. **7** **OR** Question No. **8**

Question No. **9** **OR** Question No. **10**

Question No. **11** **OR** Question No. **12**.

(2) Use of non-programmable calculator is permitted.

(3) Assume suitable data wherever necessary.

(4) Illustrate your answers wherever necessary with the help of neat sketches.

(b) Let X be the random variable giving the number of heads in three tosses of a fair coin. Find :

(i) Probability function f(x).

(ii) Distribution function F(x).

(iii) Also draw the graphs of f(x) and F(x). 7

11. (a) Find mathematical expectation of discrete random variables X whose probability function is $f(x) = (1/2)^x$, x = 1, 2, 3, ... 6

(b) Find first four moments about :

(i) Origin and

(ii) Mean for the probability distribution :

$$f(x) = \begin{cases} \dfrac{4x(9-x^2)}{81} & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$ 7

OR

12. (a) If 3% the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs (i) between 1 and 3 (ii) exactly 2 (iii) at least 2 bulbs will be defective. 6

4. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

and hence find $\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.

7

5. (a) Find z-transform of sin nθ and cos nθ. 7

(b) Find inverse z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$. 7

OR

6. (a) If $z\{f(n)\} = \bar{f}(z)$ then show that :

$$z\left\{\frac{f(n)}{n+k}\right\} = z^k \int_z^\infty \frac{\bar{f}(z)}{z^{k+1}}dz.$$

Hence find $z\left[\frac{1}{n+1}\right]$. 7

(b) Solve by z-transform :

$y_{n+2} + y_n = 2, \quad y_0 = y_1 = 0.$ 7

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7. (a) Find the modal matrix of :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ 6

(b) Solve $\frac{d^2x}{dt^2} + x = 0$; given $x(0) = 0$, $x'(0) = 1$ by matrix method. 6

(c) Find whether the vectors given below are linearly dependent or not. Also find the relation between them if possible :

$X_1 = [1, 0, 2, 1]$, $X_2 = [3, 1, 2, 1]$, $X_3 = [4, 6, 2, -4]$, $X_4 = [-6, 0, -3, -4]$. 6

OR

8. (a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$ and hence find A^{-1}. 6

(b) Use Sylvester's theorem to show that

$2 \sin A = \{\sin 2\}\, A$ where $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$. 6

(c) If $\xi = x \cos \alpha - y \sin \alpha$, $\eta = x \sin \alpha + y \cos \alpha$ then prove that the transformation is orthogonal and hence write the inverse transformation. 6

9. (a) Can the function

$$f(x) = \begin{cases} C(1-x^2) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

be a distribution function ? Explain. 7

(b) The joint probability function of two discrete random variables X and Y is given by :

$$f(x, y) = \begin{cases} C\,xy & x = 1, 2, 3 \text{ and } y = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) $P(1 \le X \le 2, Y \le 3)$.

(ii) Marginal probability function of X and Y.

(iii) Determine whether X and Y are independent. 7

OR

10. (a) Let $f(x, y) = \begin{cases} e^{-(x+y)} & x \ge 0, y \ge 0 \\ 0 & \text{otherwise} \end{cases}$

be the joint density function of X and Y. Find

(i) Marginal density function of X and Y.

(ii) Conditional density function of X given Y. 7

1. (a) If $L[f(t)] = \bar{f}(s)$ then show that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)\, ds$.

Hence find $\int_0^\infty e^{-t}\left[\frac{\cos 6t - \cos 4t}{t}\right] dt$. 7

(b) Using convolution theorem, find $L^{-1}\left[\frac{s^2}{\left(s^2 + a^2\right)^2}\right]$. 7

OR

2. (a) Solve $\frac{d^2y}{dt^2} + y = u(t - \pi) - u(t - 2\pi)$ given that $y(0) = y'(0) = 0$. 7

(b) Find the Laplace transform of the periodic function $f(t) = \frac{kt}{T}$ for $0 < t < T$ and $f(t + T) = f(t)$. 7

3. If $f(x) = e^{-\beta x}$, using Fourier integral show that

$$\frac{p}{2} e^{-\beta x} = \int_0^\infty \frac{? \sin ?x}{\beta^2 + ?^2} d?$$, for $\beta > 0$. 7

OR

(b) Let X and Y have joint density function

$$f(x, y) = \begin{cases} x + y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find conditional expectation and variance of Y given X. 7